Lightning Surge Analysis by EMTP and Numerical Electromagnetic Analysis Method

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Abstract - This paper summarizes basic assumption, related problems and the application limit of the well-known EMTP for a lightning surge analysis. As an alternative to the EMTP, numerical electromagnetic analysis (NEA) methods are briefly explained and application examples are demonstrated. Also, a difference between the EMTP and FDTD simulations is discussed. Because the NEA methods solves Maxwell’s equation directly with no assumption of a wave propagation mode, the NEA methods are becoming a very powerful approach for the lightning surge analysis.

Keywords: lightning surge, EMTP, numerical electromagnetic analysis

I. Introduction

It is not necessary to explain the significance of a lightning surge analysis in a power system. For the last 30 years, the electromagnetic transients program (EMTP) originally developed by the Bonnevill Power Administration, US Department of Energy [1, 2], has been the most powerful tool to carry out a predictive calculation of lightning overvoltages for insulation design and coordination of a substation and transmission lines. However, it is well-known that the EMTP is based on a circuit theory assuming TEM mode wave propagation, and the parameters of a circuit are to be given for an EMTP simulation [3].

Recently, digital control circuits have become quite common in power stations / substations and in intelligent buildings, and a number of electromagnetic disturbances due to lightning and switching operation have been reported [4]. Also, traveling wave propagation of a partial discharge on a gas-insulated bus [5] and a detailed analysis of archon flashover on a transmission tower [6] become significant for a more reliable operation and insulation coordination. These phenomena involve non-TEM mode propagation in a very high frequency range, and can not be analyzed by a conventional circuit-theory based approach such as the EMTP. Because of the above explained situation, numerical electromagnetic analysis methods are becoming an alternative but a very effective approach to analyze a lightning surge [7, 8].

This paper summarizes basic assumptions and related problems of the EMTP and thus explains the application limit of the EMTP for a lightning surge analysis. Then, a brief summary of a numerical electromagnetic analysis (NEA) methods is presented together with application examples. Finally, a difference between the EMTP and the FDTD simulations is investigated.

II. Recommended Modeling Method by EMTP [3]

The EMTP is based on a circuit theory which is derived from TEM mode wave propagation. Therefore, it should be noted that the EMTP is not applicable to a phenomenon associated with non-TEM mode propagation. Also, all the
parameters of a circuit are to be prepared. Fig. 1 illustrates a basic configuration of a model system recommended in Japan for a lightning surge simulation by EMTP.

A. Number of towers

Five towers from a substation gantry are to be considered, because traveling waves reflected from the towers affect a lightning surge voltage in a substation in Fig. 1. The first reflection, when lightning strikes the first tower at \( t = 0 \), from the last tower arrives at the substation entrance after two travel times \( T \) between the first and the last towers.

For example, \( x_s = 300 \text{m} \) with light velocity \( c_s = 300 \text{m/µs} \) gives the time \( T = 10 \text{µs} \). Thus, a simulation result is accurate up to \( 10 \text{µs} \), which is enough in general to observe the maximum overvoltage and the time to the peak.

B. AC sources and matching termination

The left side of the last tower is represented by a multiphase matching impedance (resistance matrix), which is given as a characteristic impedance matrix including a mutual impedance of the transmission line at a dominant transient frequency \( f \) defined by [9]:

\[
\begin{align*}
\tau &= 1/4\pi : \text{open-circuited line} \\
\tau &= 1/2\pi : \text{short-circuited line} \\
\tau &= 1/3\pi : \text{matching termination line}
\end{align*}
\]

(1)

where \( \tau = x_0/c_s \), \( x_0 = 5x_t + x_s \): total line length

An ac voltage source is connected to the other side of the matching impedance as illustrated in Fig. 1 to take into account the effect of the ac steady state voltage on a lightning surge.

C. Lightning current and impedance

A lightning current is represented by a ramp waveform with the wavefront duration \( T_f = 1 \text{µs} \) and the wavetail \( T_T = 70 \text{µs} \). Occasionally a concave current of which the waveform is defined in the following equation is adopted in Japan to represent a real lightning current more accurately [10], [11].

\[
I(t) = I_0 [1 - \cos(\alpha t)] \quad \text{for} \quad t \leq T_f
\]

(2)

\[
= \text{linearly decreasing function for} \quad t \geq T_f
\]

where \( I_0 \) : peak current, \( \alpha = \pi / 2T_f \), i.e. \( f = 1/4T_f \).

The recommended value of the lightning current amplitude is given in Table 1 for various transmission voltage in Japan.

The impedance of a lightning path is represented as a parallel resistance to a current source as illustrated in Fig. 1. The resistance value is taken to be 400Ω which was derived by Bewley [12].

D. Tower and gantry

A transmission tower is represented by four distributed-parameter lines [13] as illustrated in Fig. 2, where

\[
Z_1 : \text{tower top to the upper phase arm} = \text{upper to middle} = \text{middle to lower},
\]

\[
Z_2 : \text{lower to tower bottom}
\]

Table 1 gives a typical value of the surge impedance.

The propagation velocity \( c \) of a traveling wave along a tower is taken to be:

\[
c_s = 300 \text{ m/µs} \quad \text{light velocity in free space}
\]

To represent traveling wave attenuation and distortion, an \( RL \) parallel circuit is added to each part as illustrated in Fig. 2.

2. The values of the \( R \) and \( L \) are defined in the following equation.

\[
R = \Delta R/\tau, \quad L = 2\tau R
\]

\[
\Delta R = \Delta R_1 = \Delta R_2 = 2Z_0 \cdot \ln(1/\alpha)/h - x_0)
\]

\[
\Delta R = 2Z_0 \cdot \ln(1/\alpha)/h
\]

(4)

where \( \tau = h/c_s \) : traveling time along the tower

\[
\alpha_s = 0.89 \quad \text{attenuation along the tower}
\]

\[ h \] : tower height

A substation gantry is represented by a single distributed line with no loss.

E. Tower footing impedance

A tower footing impedance is suggested in Japan to be modeled as a simple linear resistance \( R_t \), although a current-dependent nonlinear resistance is recommended by the IEEE and the CIGRE [14]-[17] and the inductive and capacitive characteristics of the footing impedance are well-known. A recommended value of the resistance for each voltage class is given in Table 1.

F. Archorn flashover

An archorn flashover is represented either by a piecewise linear inductance model with time controlled switches as illustrated in Fig. 3(a) or by a nonlinear inductance in Fig. 3(b) based on a leader progression model [18], [19]. The parameters \( L_i \) \((i = 1 \text{ to } 3)\) and \( t_{i-1,i} \) assuming the initial time \( t_0 = 0 \) in Fig. 3(a) are determined from a measured result of the
V-I characteristic of an archorn flashover. Then the first EMTP simulation with no archorn flashover is carried out in Fig. 1, and the first flashover phase and the initial time \( t_0 \) are determined from the simulation results of the voltage waveforms across all the archorns. By adopting the above parameters, the second EMTP simulation only with the first phase flashover is carried out to determine the second flashover phase. By repeating the above procedure until no flashover occurs, a lightning surge simulation by the piecewise linear model is completed. Thus, a number of pre-calculations are necessary in the case of multiphase flashovers, while the nonlinear inductance model needs no pre-calculation and is easily applied to multiphase flashovers. The detail of the leader progression model is explained in Reference [18], and that of the nonlinear inductance model in Reference [19].

G. Transmission line

Most transmission lines in Japan are of twin-circuit vertical configuration with two ground wires, and thus are composed of eight conductors. It is recommended to represent the line by a frequency-dependent model of the EMTP. But, a distributed-line model with a fixed propagation velocity, attenuation and surge impedance, i.e. fixed-parameter distributed-line model explained in Sec. 4.2.2.4 of Reference [2], is often used, and is, in general, good enough.

H. Corona wave deformation

Japanese guideline neglects corona wave deformation, although it is taken into account in the CIGRE and the IEEE guidelines [14]-[17]. The reason for neglecting the corona wave deformation is that the resultant evervoltage is in the safety side in general.

I. Arrester

An inductance of a lead wire and the arrester itself is connected in series to the arrester model, because it affects a transient voltage and current of the arrester. Also, a capacitance of the arrester is considered if necessary. To represent a very fast impulse characteristic of the arrester, an IEEE model [20] or a model of a nonlinear resistance with a nonlinear inductance [21] is occasionally adopted. The latter model takes into account the hysteresis of an arrester V-I characteristic by adding the nonlinear inductance to the nonlinear resistance in series.

J. Substation

(1) Gas insulated bus and cable

A cable and a gas-insulated bus are represented either as three single-phase distributed lines with its coaxial mode surge impedance and propagation velocity or as a three-phase distributed line system. For a gas-insulated substation involves quite a number of gas-insulated buses/lines, the pipes are, in most cases, eliminated by assuming the voltage being zero.

(2) Circuit breaker, disconnector, transformer, bushing

A circuit breaker (CB) and a disconnector are represented by lumped capacitances between the poles and to the earth. A transformer is also represented by a capacitance to the earth unless a transferred surge to the secondary circuit is needed to be calculated. A bushing is represented by a capacitance. Occasionally it is represented by a distributed line.

(3) Grounding mesh

A grounding mesh is in general not considered in a lightning surge simulation, and is regarded as a zero-potential surface. When dealing with an incoming surge to a low-voltage control circuit, the transient voltage of the grounding mesh should not be assumed zero, and its representation becomes an important but difficult subject [22].
III. Applicable Limits and Problems of EMTP

A. Lightning current and impedance

(1) Lightning current waveform

Advanced technologies of measuring a lightning performance has revealed that a lightning current waveform is not as simple as a double exponential wave and a ramp wave [23], [24]. Unless a measured waveform or a recommended waveform is given, the EMTP can not handle a lightning current.

(2) Lightning path impedance

The lightning-path impedance of 400Ω in Fig. 1 was derived by Bewley [12], but the value seems not correct, because the lightning velocity is assumed equal to the light velocity in free space. On the contrary, Diesendorf suggested the value as 1000 to 2000 Ω[25].

The impedance value of a real lightning path has not been made clear, and requires further investigation.

B. AC source voltage

An ac source voltage is often neglected in a lightning surge simulation. It, however, has been found that the ac source voltage affects a flashover phase of an archorn especially in the case of a rather small lightning current. Fig. 4 is a measured result of archorn flashover phases as a function of the ac source voltage on a 77kV transmission line in Japan for a summer [26]. The measurements were carried out in two 77 kV substations by surge recorders installed in the substations. From the recorded voltages and currents, Fig. 4 was obtained. The figure clearly shows that the archorn flashover phase is quite dependent on the ac source voltage, i.e. a flashover occurs at a phase of which the ac voltage is in the opposite polarity of a lightning current. Table 2 shows a simulation result of archorn peak voltages (archorn not operating) on (a) the 77kV line and (b) a 500kV line [27]. The simulation was carried out in a similar circuit to Fig. 1, but another five towers were added instead of the gantry and the substation.

The parameters are the same as those in Table 1 for a 77 kV system except the lightning current of 40 kA based on the field measurement[26]. The lower phase archorn voltage is relatively smaller than the other phase archorn voltages on the 500kV line compared with those on the 77kV line. Thus, an archorn flashover phase on an EHV line is rather independent from the ac source voltage, and the lower phase flashover is less probable than the other phase flashover. On the contrary, flashover probability is rather same on each phase and a flashover is dependent on the ac source voltage on a low voltage line.

C. Tower model

(1) Problem of recommended tower model

Fig. 5 shows simulation results of archorn flashover phases by a simple distributed line “tower model” i.e. neglecting the RL circuit in Fig. 2 with the parameters in Table1, and by the recommended model illustrated in Fig. 2. The simulation circuit is the same as that described for Table 2 in the previous section. This figure should be compared with the field test result shown in Fig. 4. It is clear that the recommended model can not duplicate the field test result, while the simple

<table>
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<td>500kV</td>
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<td>4332 / 1.025</td>
</tr>
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<td>middle</td>
<td>820.2 / 1.024</td>
<td>4334 / 1.025</td>
</tr>
<tr>
<td>lower</td>
<td>720.0 / 1.035</td>
<td>3423 / 1.122</td>
</tr>
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</table>

Fig. 4  Measured results of archorn flashover phases on a 77kV transmission line
* single-phase FO, × two-phase FO, ○ three-phase FO
distributed line model shows a good agreement with the field test result. The reason for the poor accuracy of the recommended model is that the model was developed originally for a 500kV line on which the lower phase flashover was less probable as explained in the previous section. Thus, the recommended tower model tends to result in lower flashover probability of the lower phase archorn. An R-L parallel circuit between two distributed lines in Fig. 2 represents traveling wave attenuation and distortion along a tower. The $R$ and $L$ values were determined originally based on a field measurement ($\alpha$ in eq. (4)), and thus those are correct only for the tower on which the measurement was carried out. Sometimes, the R-L circuit generates unreal high frequency oscillation. This indicated a necessity of further investigation of the R-L circuit.

(2) Impedance and admittance formulas

A number of tower models have been proposed, but most of them are not general, i.e. a tower model shows a good agreement with a specific case explained in the paper where the model is proposed. The following IEEE/CIGRE formula of the tower surge impedance is well-known and is widely adopted in a lightning surge simulation [15], [17].

$$Z_t = 60 \ln(\cot(0.5 \tan^{-1}(R/h)))$$

(5)

where $R = (r_1h + r_2h + r_3h)/h$: equivalent radius of the tower represented by a truncated cone, $h = h_1 + h_2$:

$r_1$, $r_2$, $r_3$: tower top, midsection and base radii [m]

$h_1$: height from midsection to top [m]

$h_2$: height from base to midsection [m]

When the tower is not a cone but a cylinder, then the above equation is rewritten by:

$$Z_t = 60 \ln(h/r)$$

(6)

where $r$: radius of a cylinder representing a tower

Table 3 compares various tower models (surge impedance) with measured results [28]. As is clear from the average error to the measured results given at the bottom of the table, Ametani’s formula shows the highest accuracy [28]. Hara’s empirical formula also shows a quite high accuracy [29]. The IEEE/CIGRE model shows a rather poor accuracy.

The recommended value of a tower surge impedance for each voltage class in Table 1 was determined by field measurements in Japan. Although the surge impedance is a representative value, it can not be applied to every tower as is clear in Table 3.

Wave deformation on tower structures (L- or T-shape iron conductor) can be included in a lightning surge analysis, if required, based on the approach in Reference [30].

(3) Frequency-dependent effect of a tower

The frequency-dependent effect of a tower is readily taken into account in a transient simulation by combining the frequency-dependent tower impedance [28] with Semlyen’s or Marti’s line model [31], [32] in the EMTP [33].

(4) Influence of surge impedance and frequency-dependent effect

It should be pointed out that the influence of the surge impedance and the frequency-dependent effect of a tower is heavily dependent on the modeling of a tower footing impedance, which will be discussed in the following section. When the footing impedance is represented by a resistive model as recommended in Japan or by a capacitance model, then the influence of the tower surge impedance and the frequency-dependent effect of a traveling wave along the tower becomes rather noticeable. On the contrary, those cause only a minor effect when the footing impedance is represented either by an inductive model or by a nonlinear resistance. Fig. 6 shows an example [7], [33]. The measurement was carried out on a 500 kV tower by applying a current in Fig. 6(a-1) to the top of the tower. The tower top voltage predicted by a distributed-line model with a constant tower surge impedance and no R-L circuit, Fig. 6(c-1), differs from that by the frequency-dependent tower model, Fig. 6(b) which agrees with the measured result, in the case of the footing impedance being a resistance. On the contrary, in the case of an inductive footing model, the tower top voltage obtained by the distributed-line model shows a rather good agreement with the measured result. It should be also noted that some 10% variation of the tower surge impedance does not affect the result in the inductive footing impedance case.
Thus, it is concluded that the frequency-dependent effect of wave propagation along a tower can be neglected and the value of the surge impedance is not significant unless a tower footing impedance is represented by a resistive or a capacitive model.

(5) TEM mode propagation

All the above discussions are based on TEM mode propagation of an electromagnetic wave along a vertical tower. It has been pointed out in many publications that the electromagnetic wave along the tower is not the TEM mode especially at the time of lightning instance to the tower [8], [34]. The same is applied to the lightning path impedance.

D. Tower footing impedance

(1) Linear footing impedance

It has been known in general that the footing impedance tends to be capacitive in the case of a high resistivity earth, and inductive in the low resistivity earth case. A problem of the representation is: The footing impedance can be resistive, inductive and capacitive depending on the season and the weather when a measurement is made, i.e. the impedance is temperature- and soil moisture-dependent. Therefore, it is not easy to select a model of the footing impedance and this is the reason why a resistance model is adopted in Japan.

(2) Current-dependent nonlinearity

A number of papers have discussed the current-dependence of a tower footing impedance, and have proposed various models of the nonlinear footing impedance. It has been a common understanding that the current-dependence decreases a
lightning surge voltage at the tower and thus decreases a lightning overvoltage at a substation. Therefore a simulation neglecting the current dependence gives a severer overvoltage, i.e. a safer side result from the insulation design viewpoint. By this reason, again a pure resistance model is recommended in Japan.

(3) Non-uniform and frequency-dependent characteristics

All the grounding electrodes, either horizontal (counterpoise) or vertical, show a non-uniform characteristic [35], which corresponds a so-called critical length of the electrode [36]. The characteristic can be taken into account in an EMTP simulation by adopting a model circuit described in Reference [35] together with the frequency-dependent effect. However, this approach requires a measured result of the grounding electrode to be simulated.

A general solution for a transient response of a grounding electrode is rather easily obtained by a numerical electromagnetic analysis method such as a finite-difference time-domain (FDTD) method [8], [37] including non-TEM mode propagation.

E. Archon flashover model

There exist a number of archon flashover models. To investigate the accuracy, phase-wire voltages at the first tower in Fig. 1 are calculated by various archon models and are compared[38]. A switch (time-controlled) model and a flashover switch model show not satisfactory agreement especially in the wavefront with the nonlinear inductance model of which the accuracy has been confirmed to be high in comparison with an experimental result [19]. A v-t characteristic model and a piecewise linear inductance model show a reasonable accuracy except that the maximum voltage of the former is greater and that of the latter is lower than that calculated by the nonlinear inductance model. It might be noteworthy that a current and energy consumed by an arrester in a substation are dependent on an archon model. The switch and the flashover switch models result in much higher energy consumed by an arrester.

An archon flashover might be affected by transient electromagnetic coupling between a lightning path, a tower and phase conductors which are perpendicular to the tower and the lightning path. Such coupling can not be handled by the EMTP and is easily solved by a numerical electromagnetic analysis [7], [8], [39].

F. Transmission line, feeder, gas-insulated bus

(1) Frequency-dependent transmission line impedance

Although a frequency-dependent line model, i.e. Semlyen’s or Marti’s model [31], [32], is recommended, the maximum error of Marti’s model is observed to be about 15 % at the wavefront of an impulse voltage on an 1100 kV untransposed vertical twin-circuit line in comparison with a field test result [40] of which a surge waveform is shown in Fig. 7 in comparison with EMTP simulation result. The estimation of possible errors incurred by using these models in a lightning overvoltage simulation is not straightforward, because it involves a nonlinearity due to an archon flashover dependent on a lightning current, an ac source voltage, a flashover phase and so on. This is an important subject to be investigated in future.
(2) Finite length of a line and a gas-insulated bus

Carson’s and Pollaczek’s earth return impedance of an overhead line and an underground cable were derived based on the assumption of an infinitely long line on the basis of TEM mode propagation [41]-[44]. A real line is not infinitely long at all. The separation distance $x$ of a UHV/EHV transmission line between adjacent towers and the length of a gas-insulated bus are in the same order of their height $h$. If the condition, that $x$ is far greater than $h$ and $h$ is far greater than the radius $r$, is not satisfied, Carson’s and Pollaczek’s impedances are not applicable. Fig. 8 shows an example of the impedance of a finite length line evaluated by the following equation [45].

$$Z_{\text{finite}} = \frac{j\omega \mu_0}{2\pi} \left( \ln x + \frac{1}{2} \left( \frac{d_i}{x} \right)^2 \right) + x \ln \left( S_{i0} \right) \left[ \frac{S_i - S_j}{d_i} - \frac{\sqrt{x^2 + d_i^2} + \sqrt{x^2 + d_j^2} + d_i - S_j}{\sqrt{1 + \left( S_{j0}/x \right)^2}} \right]$$ \hspace{1cm} (7)

where $d_i = \sqrt{(h_i - h_j)^2 + y^2}$, $S_{i0} = \sqrt{(h_i + h_j + 2h_0)^2 + y^2}$

$y$ : horizontal separation between conductors $i$ and $j$

$h_i$, $h_j$ : height of conductors $i$ and $j$

$h_e = \sqrt{\mu_0/\omega \rho_e}$ : complex penetration depth [46]

It should be clear in Fig. 8 that Carson’s impedance assuming an infinitely long line is far greater than that of a real finite line, when $x/h$ is not greater enough than 1. The reason for this is readily understood from the following equation [43].

$$Z_{\text{Car}} = \int_0^x \int_0^x A(x_i, x_j) dx_j dx_i = \int_0^x B(x) dx$$ \hspace{1cm} (8)

$$Z_{\text{fin}} = \int_0^{x_f} \int_0^{x_f} A(x_i, x_j) dx_j dx_i = \int_0^{x_f} B'(x) dx$$ \hspace{1cm} (9)

As is clear from the above equations, $Z_{\text{Car}}$ involves mutual coupling from the infinitely long conductor “$j$”, while $Z_{\text{fin}}$ involves mutual coupling from the conductor “$j$” with the finite length $x_f$. In fact, $Z_{\text{Car}}$ becomes infinite because of the infinite length, and thus “per unit length” impedance is necessarily defined. It should be noted that the per unit length impedance $\Delta Z_{\text{Car}}$ of eq. (8) has included the mutual coupling from the infinitely long conductor “$j$”. On the contrary, $\Delta Z_{\text{fin}}$, if we define the per unit length impedance in eq. (9), includes the mutual coupling only for the finite length $x_f$. Thus,

$$\Delta Z_{\text{Car}} > \Delta Z_{\text{finite}}$$ \hspace{1cm} (10)

On the contrary, the per unit length admittance of an infinitely long line is smaller than that of a finite length line.

From the above discussion, it should now be clear that Carson’s and Pollaczek’s earth return impedances may not be applied to a lightning surge analysis, because the separation distance $x$ between adjacent towers is the same order as the line height. The same is true for a gas-insulated bus, because its length, height and radius are in the same order. This requires further work which is interesting and significant.

It is noteworthy that the propagation constants of a finite line is nearly the same as that of an infinitely long line, but the characteristic (surge) impedance is smaller, because of a smaller series impedance and a greater shunt admittance of the finite line. Furthermore the ratio of the surge impedances of two finite lines is nearly the same as that of two infinitely long lines. Finally, traveling wave reflection, refraction and deformation on the finite line is not much different from those on the infinitely long line.

(3) Feeding line from a transmission line to a substation – Inclined conductor

A feeding line from the first tower to the substation via the gantry in Fig. 1 is inclined, i.e. the height is gradually decreased. As a result, its surge impedance is also decreased gradually and thus no significant reflection of traveling waves occurs along the feeding line until the substation entrance in physical reality. Because the surge impedance (about 70$\Omega$) of a gas-insulated bus or a bushing is much smaller than that of an overhead line (300 to 500$\Omega$), noticeable reflection appears at the substation entrance, if the inclined configuration of the overhead feeding line is not considered. It is better to consider the inclined configuration of a feeding line if an accurate simulation is required. A maximum difference of 7% in a substation entrance voltage is observed when the inclined configuration is considered [43], [45].
G. Corona wave deformation

The reason for corona wave deformation being not considered in a lightning surge simulation in Japanese guideline is that a simulation result neglecting the corona is expected to be higher than that considering corona and thus the result is on a safer side from the insulation viewpoint. The possible errors incurred by ignoring corona are observed to be less than 10% when lightning strikes the first tower in Fig. 1[11]. Although sophisticated corona models have been proposed [48], [49], the reliability and stability in a lightning overvoltage simulation is not confirmed.

It is noteworthy that the corona wave deformation can result in a higher overvoltage at a substation entrance under a specific condition. Fig. 9 shows a field test result of a normalized voltage ratio $K$ for the negative polarity case defined in the following equation on a 6.6 kV line [50], [51].

$$K = \frac{V_n}{V_m}$$

where $V_n$ : maximum phase-wire (PW) voltage at the receiving end (substation entrance)

$V_m$ : normalised voltage with no corona discharge

The experiment was carried out on a 6.6 kV line with one phase wire and one ground wire which were terminated by resistances $R_p$ and $R_g$ at the remote end. An impulse voltage up to 800 kV was applied to the sending end of the ground wire. The back flashover in Fig. 9 was represented by short-circuit of the ground and phase wires. For corona wave deformation decreases a traveling wave voltage on a line, it is a common understanding that the line voltage is decreased by the corona wave deformation and thus the ratio $K$ is less than 1. In the case of no corona discharge, $K$ is nearly equal to 1 on a short distance line. It was observed that a measured result of $K$ on a single-phase line was less than 1.

Fig. 9(a) shows that $K$ in the case of no back-flashover becomes greater than 1 as the applied voltage is increased, i.e. corona discharge occurs. On the contrary in Fig. 9(b), $K$ is less than 1. The reason for the phenomena is readily explained as a result of different attenuation on a phase wire and a ground wire due to corona discharge, and negative reflection of a heavily attenuated traveling wave on the ground wire. The phenomena are less noticeable in the positive polarity case. The detail has been explained in References [50] and [51]. The phenomena have been also realized qualitatively by an EMTP simulation. The increase of a phase-wire voltage at a substation entrance is expected to be more pronounced on an EHV/UHV transmission line on which a corona discharge hardly occurs on a phase wire because of a multiple bundled conductor, while a heavy corona discharge is expected on a ground wire.

H. Phase-to-phase lightning surge

Most of the previous studies on a lightning overvoltage concerned an overvoltage to the earth. A phase-to-phase overvoltage, however, can damage insulation between phases such as core-to-core insulation in a gas-insulated bus in which three-phase cores are enclosed in the pipe. Fig. 10 shows an EMTP simulation result of an inter-phase lightning overvoltage at a substation entrance on a 77kV vertical twin-circuit line, when phases a and b flashovers. The simulation was carried out on a system composed of a 10 km 245/77kV quadruple circuit line and a 10 km 77 kV line. The 77 kV line was connected to a substation through a three-phase underground XLPE cable with the length of 500 m. Because of a lower attenuation of aerial propagation modes, the phase-to-phase overvoltage becomes greater than the phase-to-earth overvoltage especially in the case of a lightning strike to a tower far from a substation.

The phase-to-phase lightning overvoltage needs further investigation.
Table 4 Various methods of numerical electromagnetic analysis

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IV. Numerical Electromagnetic Analysis Method for Lightning Surges

It is hard to handle a transient associated with non-TEM mode propagation by conventional circuit-theory based tools such as the EMTP, because the tools are based on TEM mode propagation. To overcome the problem, a numerical electromagnetic analysis method looks most promised among existing transient analysis approaches for it solves Maxwell’s equation directly without any assumptions often made for the circuit-theory based tools.

This chapter describes the basic theory of two representative methods, i.e. method of moment (MoM) and finite-difference time-domain (FDTD) method, of the numerical electromagnetic analysis. Also, to demonstrate the usefulness and advantages, four typical examples are presented.

A. Numerical Electromagnetic Analysis Method

(1) Various method, at present

Table 4 categorized various methods of numerical electromagnetic analysis (NEA) [7, 8]. The method of moments (MoM) in the frequency and time domains [52-57], and the finite-difference time-domain (FDTD) method [58, 59], both for solving Maxwell’s equations numerically, have frequently been used in calculating surges on power systems. Applications of the finite element method (FEM) and the transmission line method (TLM) to surge calculations have been rare at present. The MoM and the FDTD method are, therefore, two representative approaches in surge calculations.

(2) Methods of Moments (MoMs) in the Time and Frequency Domains

a) MoM in the Time Domain

The MoM in the time domain [52, 53] is widely used in analyzing responses of thin-wire metallic structures to external time-varying electromagnetic fields. The entire conducting structure representing the lightning channel is modeled by a combination of cylindrical wire segments whose radii are much smaller than the wavelengths of interest. The so-called electric-field integral equation for a perfectly conducting thin wire in air as in Fig. 11, assuming that current $I$ and charge $q$ are confined to the wire axis (thin-wire approximation) and that the boundary condition on the tangential electric field on the surface of the wire (this field must be equal to zero) is fulfilled, is given by

$$\hat{s} \cdot E_{inc}(r,t) = \frac{\mu_0}{4\pi} \int_{-\infty}^{\infty} \left[ \hat{s} \cdot \hat{s}' \frac{\partial l(s',t')}{\partial t'} + e \frac{\hat{s} \cdot \vec{R}'}{R'} \frac{\partial l(s',t')}{\partial s'} - e^{-\frac{\hat{s} \cdot \vec{R}}{R}} \frac{\partial l(s',t')}{\partial s'} q(s',t') \right] ds'$$  \hspace{1cm} (12)

where $q(s',t') = -\frac{\partial l}{\partial s'} \frac{\partial l}{\partial s'} d\tau$

$C$ is an integration path along the wire axis, $E_{inc}$ denotes the incident electric field that induces current $I$, $R = r - r'$, $r$ and $t$ denote the observation location (a point on the wire surface) and time, respectively, $r'$ and $t'$ denote the source location (a point on the wire axis) and time, respectively, $s$ and $s'$ denote the distance along the wire surface at $r$ and that along the wire axis at $r'$, $\hat{s}$ and $\hat{s}'$ denote unit vectors tangent to path $C$ in (12) at $r$ and $r'$, $\mu_0$ is the permeability of vacuum, and $e$ is the speed of light. Through numerically solving (12), which is based on Maxwell’s equations, the time-dependent current distribution along the wire structure (lightning channel), excited by a lumped source, is obtained.

The thin-wire time-domain (TWTD) code [52] (available from the Lawrence Livermore National Laboratory) is based on the MoM in the time domain. One of the advantages of the use of the time-domain MoM is that it can incorporate nonlinear effects such as the lightning attachment process [54], although it does not allow lossy ground and wires buried in lossy ground to be incorporated.

b) MoM in the Frequency Domain

The MoM in the frequency domain [55] is widely used in analyzing the electromagnetic scattering by antennas and other metallic structures. In order to obtain the time-varying responses, Fourier and inverse Fourier transforms are employed. The electric-field integral equation derived for a perfectly conducting thin wire in air as in Fig. 11 in the frequency domain is given by
y=j

where \( g(r,r') = \exp \left( -\frac{jk|\mathbf{r} - \mathbf{r}'|}{\mathbf{r} - \mathbf{r}'} \right) \), \( k = \omega \sqrt{\mu_0 \varepsilon_0} \), \( \eta = \sqrt{\mu_0 / \varepsilon_0} \)

\( \omega \) is the angular frequency, \( \mu_0 \) is the permeability of vacuum, and \( \varepsilon_0 \) is the permittivity of vacuum. Other quantities in eq.(13) are the same as those in eq.(12). Current distribution along the lightning channel can be obtained numerically solving eq.(13).

This method allows lossy ground and wires in lossy ground (for example, grounding of a tall strike object) to be incorporated into the model. The commercially available numerical electromagnetic codes [56], [57], are based on the MoM in the frequency domain.

(3) Finite-Difference Time-Domain (FDTD) Method

The FDTD method [58] employs a simple way to discretize Maxwell’s equations in differential form. In the Cartesian coordinate system, it requires discretization of the entire space of interest into small cubic or rectangular-parallelepiped cells. Cells for specifying or computing electric field (electric field cells) and magnetic field cells are placed relative to each other as shown in Fig. 12. Electric and magnetic fields of the cells are calculated using the discretized Maxwell’s equations given below.

\[
E_x^{n+\frac{1}{2}}(i,j,k+\frac{1}{2}) = \frac{1 - \sigma(i,j,k+1/2)\Delta t}{1+\sigma(i,j,k+1/2)\Delta t} \left\{ \frac{2\varepsilon(i,j,k+1/2)}{2\varepsilon(i,j,k+1/2)} \right\} E_x^{n-\frac{1}{2}}(i,j,k+\frac{1}{2}) + \frac{\Delta t}{\mu(i,j,k+1/2)\Delta y\Delta z} \left[ H_y^{n+\frac{1}{2}}(i+1/2,j,k+1/2)\Delta y - H_y^{n-\frac{1}{2}}(i-1/2,j,k+1/2)\Delta y \right. \\
\left. - H_x^{n+\frac{1}{2}}(i,j+1/2,k+1/2)\Delta x + H_x^{n-\frac{1}{2}}(i,j-1/2,k+1/2)\Delta x \right]
\]

\[
H_x^{n+\frac{1}{2}}(i,j,k+\frac{1}{2}) = H_x^{n+\frac{1}{2}}(i,j-\frac{1}{2},k+\frac{1}{2}) + \frac{\Delta t}{\mu(i,j-1/2,k+1/2)\Delta y\Delta z} \left[ -E_x^n(i,j,k+1/2)\Delta x + E_x^n(i,j-1/2,k+1/2)\Delta x \right.
\left. + E_y^n(i,j-1/2,k+1)\Delta y - E_y^n(i,j-1/2,k)\Delta y \right]
\]

Equation (14), which is based on Ampere’s law, is an equation updating \( z \) component of electric field, \( E_z(i,j,k+1/2) \), at point \( x=i\Delta x, y=j\Delta y, \) and \( z=(k+1/2)\Delta z, \) and at time \( t=n\Delta t \). Eq. (15), which is based on Faraday’s law, is an equation updating \( x \) component of magnetic field, \( H_x(i,j-1/2,k+1/2) \), at point \( x=i\Delta x, y=(j-1/2)\Delta y, \) and \( z=(k+1/2)\Delta z, \) and at time \( t=n\Delta t \). Equations updating \( x \) and \( y \) components of electric field, and \( y \) and \( z \) components of magnetic field can be written in a similar manner. Note that \( \sigma(i,j,k+1/2) \) and \( \sigma(i,j,k+1/2) \) are the conductivity and permeability at point \( x=i\Delta x, y=(j-1/2)\Delta y, \) and \( z=(k+1/2)\Delta z, \) respectively, \( \mu(i,j-1/2,k+1/2) \) is the permeability at point \( x=i\Delta x, y=(j-1/2)\Delta y, \) and \( z=(k+1/2)\Delta z. \) By updating electric and magnetic fields at every point using eq.(14) and (15), transient fields throughout the computational domain are obtained. Since the material constants of each cell can be specified individually, a complex inhomogeneous medium can be analyzed easily.

In order to analyze fields in unbounded space, an absorbing boundary condition has to be set on each plane which limits the space to be analyzed, so as to avoid reflections there. The FDTD method allows one to incorporate wires buried in lossy ground, such as strike-object grounding electrodes [59], and nonlinear effects.
Fig. 12 Placement of electric-field and magnetic-field cells for solving discretized Maxwell’s equations using the FDTD method.

B. Application Examples

(1) A transient response on a grounding electrode

The impedance and admittance of a given electrical circuit are essential to analyze its steady and transient characteristics by a circuit-theory based approach such as the Electromagnetic Transients Program (EMTP) [1, 2]. Sunde’s formula of the admittance of a grounding electrode [60] is well-known and has been widely used in the world. However, the formula is only for a steady state. Sunde also proposed impedance and admittance formulas for a transient, but those require iterative calculations and the accuracy is found not satisfactory enough [61].

An electromagnetic interference due to mutual coupling between a grounding mesh and a control cable becomes a significant subject in power stations and substations [4, 62-64]. To analyze this problem, a transient impedance and admittance are indispensable. Unfortunately no formula is available, and numerical identification from a measured result looks only a promised method presently as far as the circuit-theory based approach concerns, although many grounding electrode models have been proposed [65]. On the contrary, an NEA approach requires no impedance and admittance, because those are evaluated as a part of an NEA calculation.

Fig.13 (a) illustrates the geometrical configuration of a tested grounding electrode and the experimental circuit, where only geometrical and physical parameters are required in the NEA calculation [37]. Fig.13 (b) is a comparison of an FDTD simulation result with the measured one. A satisfactory accuracy of the FDTD method is confirmed from the results. This example shows that the numerical electromagnetic analysis can solve a problem of which the impedance and admittance are not known, for the method requires no circuit parameter. Also, the mode of wave propagation may not be TEM, while the circuit-theory based approach is restricted only for the TEM propagation. Also, it should be noted that the phenomenon is three-dimensional as is clear from Fig.13 (a).

(2) Partial-discharge pulse propagation in a gas-insulated bus

Fig.14 (a) presents the geometrical configuration of a gas-insulated bus in which a pulse is generated due to a partial discharge. It should be clear in the figure that a part of the conductor is perpendicular to the remaining part. Such a conductor can not be handled by the EMTP. Furthermore, the phenomenon in this system involves a radial wave propagation other than axial one. Fig.14 (b) shows a simulation result by MoM, which reproduces the reflection from the corner of the
bus due to electromagnetic wave scattering. The scattering at the corner cannot be simulated by a circuit-theory based method. The approach is applied to develop life estimation of a power apparatus [5].

(3) Step response of a transmission tower for lightning overvoltage studies

Fig. 15(a) illustrates the configuration of a pulse test for obtaining the step response of a 500 kV transmission tower for lightning overvoltage studies. Fig. 15(b) shows the measured result of the pulse test, where a voltage-rise waveform at the tower top when a step current is injected into the tower top is measured. Fig. 15(c) shows the corresponding simulation result by FDTD method. The calculated waveforms closely reproduce the measured ones. In this problem, the tower in Fig. 15(a) is modeled three-dimensionally [7, 59]. It is hard to analyze a three-dimensional phenomenon by a conventional circuit-theory approach. Also, the transient at the wavefront might involve non-TEM coupling within the tower structures which will be demonstrated in the next example.

V. Comparison of EMTP and NEA Simulations

The theory and simulation results of the EMTP and numerical electromagnetic analysis methods (NEA) have been explained in the previous sections. A comparison of simulation results by the both methods will be presented in this section.

A. Transient Responses of a Grounding Electrode

Fig. 16 illustrates a model circuit of a grounding electrode for an EMTP simulation of which the parameters are given in the following equations [22].

\[ C_i = C_\theta - C_0, \quad G_i = G_\theta, \quad C_2 = nC_1 \]

\[ C_i = \pi \varepsilon_o x / A, \quad G_i = \pi x / \rho_c A, \quad A = \epsilon n(2x / \varepsilon \sqrt{2d \cdot r_1}) \]  

\[ C_\theta = 2\pi \varepsilon_o / \epsilon n(r_2 / r_1) \]

where \( r_1 \): radius of a bare conductor
d: buried depth of the conductor
$r_2$: $r_1+\Delta$: radius of an artificial outer insulator
$\varepsilon_r$: earth permittivity,  $\varepsilon_0$: free space permittivity
$\rho_e$: earth resistivity,  $e=2.718...$,  $n=5$

Cs and Gs in the above equation is well-known Sunde’s formula [60] of a steady-state capacitance and conductance of a horizontal grounding electrode.

Fig. 17 shows a comparison of EMTP and FDTD simulation results with a measured result [66]. The simulation results in Fig. 17 show a reasonable agreement with the measured result.

It should be noted that the EMTP simulation result is quite dependent on the parameters adopted in the simulation which is a function of geometrical and physical constants of a conductor as is clear in eq (16). On the contrary, an FDTD simulation depends very much on the analytical space, absorbing boundary, cell size and time step. The above observation has indicated that the EMTP has been numerically completed quite well, while the FDTD requires a further improvement of its numerical stability.

B. Transient responses of a tower

Fig. 18 illustrates an experimental circuit of a gas tower system, which is an 1/30th scale model of a real system, and a measured result [67]. The circuit is the same as that of a wind generation tower if there is no pipeline connected to the gas tower, and also the same as that of a transmission tower if the tower is represented by a cylindrical conductor [28] and ground and phase wires are added.

(1) EMTP Simulation

The tower in Fig. 18 is represented as a distributed-parameter line with a surge impedance $Z_0$ and a propagation velocity $c$ of which the values are evaluated by the impedance and the admittance formulas derived in Reference [28]. Fig. 19 shows a simulation result by the EMTP.

(2) FDTD simulation

Fig. 20 shows a simulation result by the FDTD.

It is observed that the simulation results in Fig.19 by the EMTP and in Fig. 20 by the FDTD agree reasonably well with the measured result in Fig. 19. A difference observed between the measured and the EMTP simulation results is estimated due to mutual coupling between the tower, the pipeline and measuring wires. Also, the frequency-dependent effect of the conductor affects the difference. A difference between the measured and the FDTD simulation results seems to be caused by a perfect conductor assumption of the FDTD method.

C. Archone voltage during a back-flashover

The electromagnetic field around a transmission tower hit by lightning changes dynamically while electromagnetic waves make several round-trips between a shield wire and the ground. During this interval, the waveforms of archorn voltages vary complexly. For a tall structure such as an EHV twin-circuit tower, the contribution of the tower surge characteristic to the archorn voltages becomes dominant because the travel time of a surge along the tower is comparable to the rise time of a lightning current. Particularly in the case of a back-flashover at such a tall tower, a powerful electromagnetic impulse is produced since the archorn voltage of several MV is chopped steeply. The electromagnetic impulse expands spherically and couples with the other phase lines. Such electromagnetic coupling is different from the TEM coupling and it may influence significantly the archorn voltages of other phases. This issue, however, has been paid little attention in analyzing a multiphase back-flashover.
**Fig. 18** Experimental setup: $R_p = 150 \Omega$

![Experimental setup diagram](image)

**Fig. 19** Measured EMTP simulation result

(a) Tower top

(b) Pipe sending end

(c) Pipe receiving end

(d) Control line receiving end

**Fig. 20** FDTD simulation results

(a) Tower top

(b) Pipe sending end

(c) Pipe receiving end

(d) Control line receiving end
To analyze such a very-fast transient electromagnetic field around a three-dimensional conductor system, electromagnetic modeling codes are appropriate. Among many available codes, the Thin-Wire Time-Domain Analysis (TWTDA) code \cite{52, 68} based on the method of moments \cite{53} is chosen in the present work, for this code allows to incorporate nonlinear effects into the analysis \cite{6}.

In this section, archorn voltages of a simulated 500 kV twin-circuit tower in Fig.21 (a) hit by lightning, in the case of one-phase back-flashover, are analyzed by a modified TWTDA code that includes a recently proposed flashover model \cite{69, 70}. A similar analysis is also carried out by EMTP \cite{1}, and the results are compared with those computed by the modified TWTDA code.

Fig.21 (b) shows measured waveforms of the voltage of a 3 m gap and the current flowing through it \cite{2}, and those computed with the TWTDA code including Motoyama’s flashover model. (1) Voltage. (2) Current.

Fig.21 Archorn voltages during a back-flashover

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Fig.21 (b) shows measured waveforms of the voltage of a 3 m gap and the current flowing through it \cite{2}, and those computed with the TWTDA code. Fig.21 (c) are the archorn voltages computed by (1)TWTDA and by (2)EMTP, in the case of a middle-phase back-flashover. (150 kA, 1.0 µs ramp current injection)

VI. Conclusion

This paper has presented a lightning surge analysis by the EMTP and by numerical electromagnetic analysis methods. Because the EMTP is based on a circuit theory assuming TEM mode propagation, it can not give an accurate solution for a high frequency transient which involves non-TEM mode propagation. Also, the EMTP can not deal with a circuit of which the parameters are not known.

On the contrary, a numerical electromagnetic analysis method can deal with a transient associated with both TEM and non-TEM mode propagation. Furthermore, it requires not circuit parameter but geometrical and physical parameters of a given system. However, it other results in numerical instability if the analytical space, the boundary conditions, the cell size etc are not appropriate. Also, it requires a large amount of computer resources, and existing codes are not general enough to deal with various type of transients especially in a large network.

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VII. References

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