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Theoretical methods for calculating electromagnetic fields from lightning discharge

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Outline

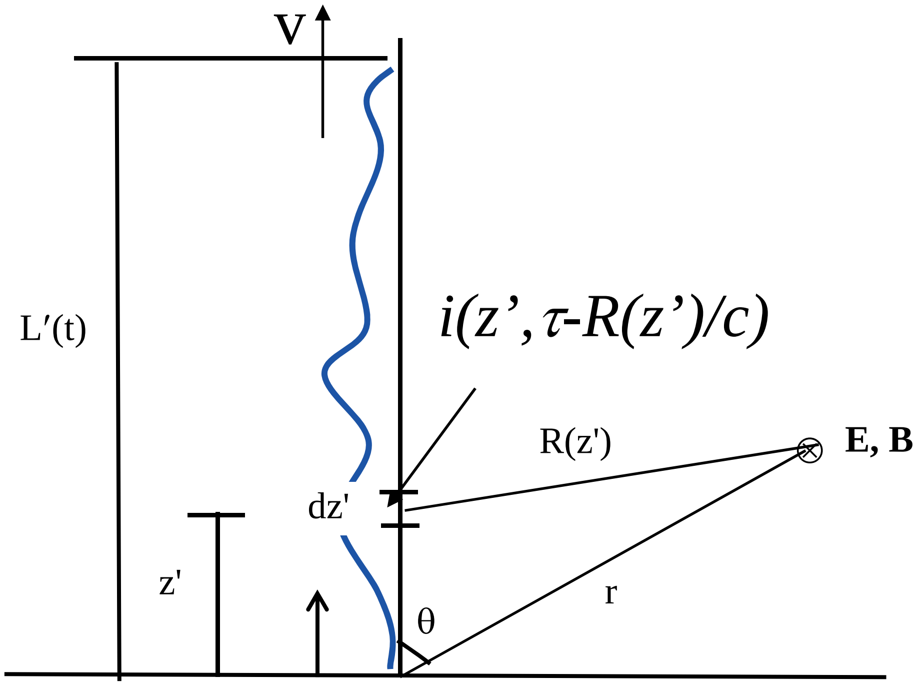
- Description of the problem
- Three different methods for field calculations
  - Dipole and monopole methods for field calculations
  - Non-uniqueness of field components
  - Fields in terms of apparent charge (third method)
- Special case of return stroke with speed of light

Acknowledgement: V.A Rakov and M. Uman, University of Florida, Gainesville

Description of the problem

- Lightning return stroke speed 1-2x10^8 m/s
- Current rise time < 10^{-6} s
- Distributed source fast changing in both space and time
- Methods of finding exact expressions for remote electric and magnetic fields

$$L'(t) < v.t$$



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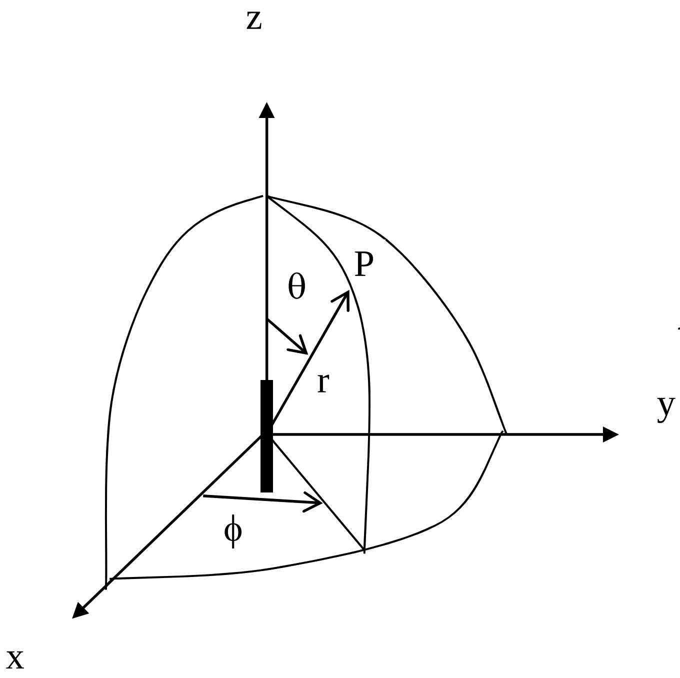
Electric fields from a dipole



Static

$$E_r = \frac{Z_0}{2\pi} m_E \cos\theta \left(\frac{1}{r^2} + \frac{1}{j\beta r^3} \right) e^{-j(\beta r - \omega t)}$$

$$E_\theta = \frac{-jZ_0\beta}{4\pi} m_E \sin\theta \left(\frac{1}{r} + \frac{1}{j\beta r^2} - \frac{1}{\beta^2 r^3} \right) e^{-j(\beta r - \omega t)}$$

$$H_\phi = \frac{j\beta}{4\pi} m_E \sin\theta \left(\frac{1}{r} + \frac{1}{j\beta r^2} \right) e^{-j(\beta r - \omega t)}$$

Radiation

Induction or intermediate

Are the field components unique?

Electric fields from line sources

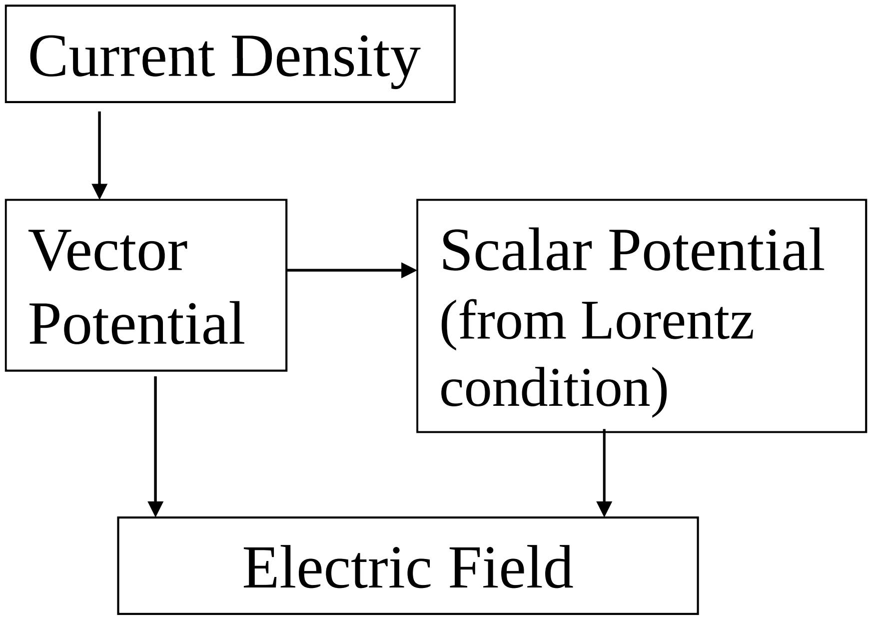
- (Line source – several dipoles connected end to end)

- Is it possible to define static, induction and radiation components uniquely?

- How do we define static field?
  - 1/distance^3 ?
  - The part of electric field given by the gradient of the scalar potential?

- How do we define radiation field?
  - 1/distance ?
  - Associated with rate of change of current, accelerating charges?
  - Rate of change of vector potential?

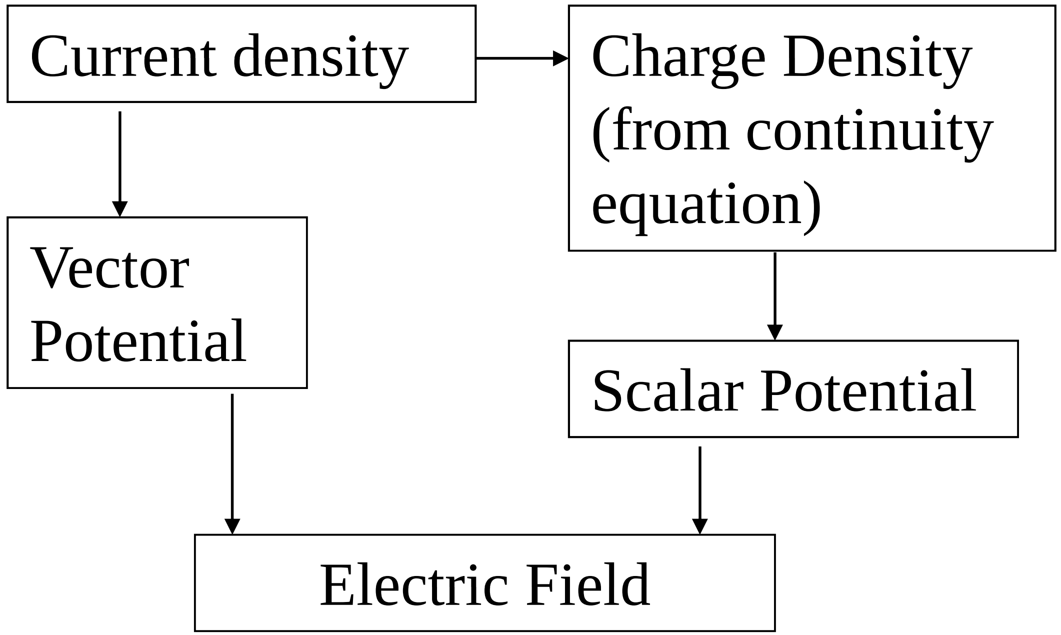
Two methods for finding electric fields



Dipole method

(Explicit use of Lorentz condition)

Monopole method



(Explicit use of continuity equation)

(Jefimenko)

Rubinstein et al. (1989) showed that for a extending step pulse, both give identical electric fields numerically.

Safaeinili and Mina [1991] showed that for extending step pulse, both expressions are analytically equivalent.

Thottappillil and Rakov (2001) showed analytically that for any distribution of current and charges on a lightning channel, the dipole and monopole methods give the same electric fields

Dipole and monopole methods for field calculations - 1

- Dipole method

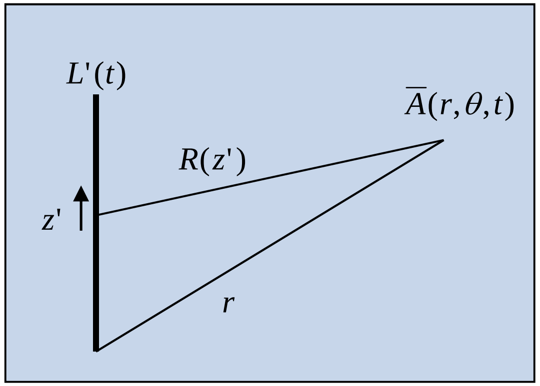
(Explicit use of Lorentz condition to find scalar potential from vector potential)

$$\overline{A}(r,\theta,\tau) = \frac{1}{4\pi\varepsilon_0 c^2}\int_0^{L'(\tau)} \frac{i(z',\tau - R(z')/c)}{R(z')}\hat{z}\,dz'$$

(Lorentz condition)

$$\nabla \bullet \overline{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$$

$$\phi(r,\theta,t) = -c^2 \int_{r/c}^{t} \nabla \bullet \overline{A}\, d\tau$$



$$\overline{E} = -\nabla\phi - \frac{\partial \overline{A}}{\partial t}$$

$$\overline{B} = \nabla \times \overline{A}$$

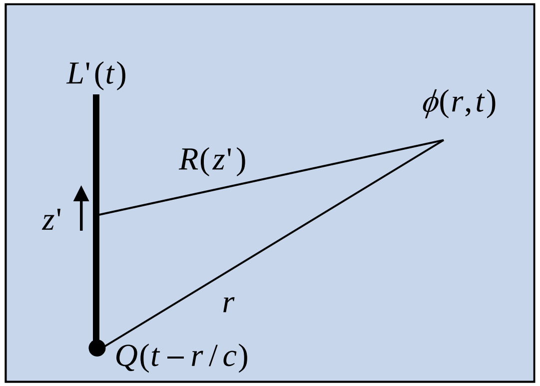
Monopole method for field calculations - 2

- The monopole method

(The continuity equation explicitly used to find charge density from current)

$$\overline{A}(r,\theta,\tau) = \frac{1}{4\pi\varepsilon_0 c^2} \int_0^{L'(\tau)} \frac{i(z',\tau - R(z')/c)}{R(z')} \hat{z}\, dz'$$

$$\frac{\partial \rho^*(z',t-R(z')/c)}{\partial t} = - \left. \frac{\partial i(z',t-R(z')/c)}{\partial z'} \right|_{t-R(z')/c=const.}$$



$$\phi(r,t) = \frac{1}{4\pi\varepsilon_0} \frac{Q(t-r/c)}{r} + \frac{1}{4\pi\varepsilon_0} \int_0^{L'(t)} \frac{1}{R(z')} \rho^*(z',t-R(z')/c)\, dz'$$

$$\overline{E} = -\nabla\phi - \frac{\partial \overline{A}}{\partial t}$$

Expressions for scalar potential

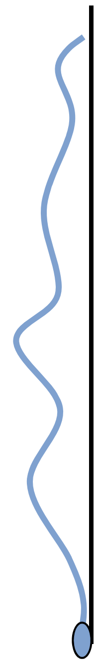
Dipole method

$$\phi(r,\theta,t) \quad = \quad \frac{1}{4\pi\varepsilon_0}\int_0^{L'(t)}\left[\frac{z'}{R^3}\int_{z'/c+R/c}^{t} i(0,\tau-R/c)d\tau+\frac{z'}{cR^2}i(0,t-R/c)\right]dz'$$

Monopole method

$$\phi(r,t) \quad = \quad \frac{1}{4\pi\varepsilon_0}\frac{Q(t-r/c)}{r}+\frac{1}{4\pi\varepsilon_0}\int_0^{L'(t)}\frac{1}{R(z')}\rho^*(z',t-R(z')/c)dz',$$

$$Q(t-r/c)=-\int_{r/c}^{t} i(0,\tau-r/c)d\tau$$



Both are analytically equivalent

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Sample calculation using the two methods
(lightning return stroke field at ground)

$$i(z',t) = i(0, t - z'/v)$$

Dipole method

$$E_V(r,t) = \frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{2-3\sin^2\alpha(z')}{R^3(z')}\int_{t_b}^{t} i\,(z', \tau - R(z')/c)d\tau\, dz'$$

$$+\frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{2-3\sin^2\alpha(z')}{cR^2(z')} i(z', t - R(z')/c)\, dz'$$

$$-\frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{\sin^2\alpha(z')}{c^2R(z')}\frac{\partial i(z', t - R(z')/c)}{\partial t} dz'$$

Monopole method

$$E_V(r,t) = -\frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{z'}{R^3(z')}\rho^*(z', t - R(z')/c)\, dz'$$

$$-\frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{z'}{cR^2(z')}\frac{\partial \rho^*(z', t - R(z')/c)}{\partial t} dz'$$

$$-\frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{1}{c^2R(z')}\frac{\partial i(z', t - R(z')/c)}{\partial t} dz'$$

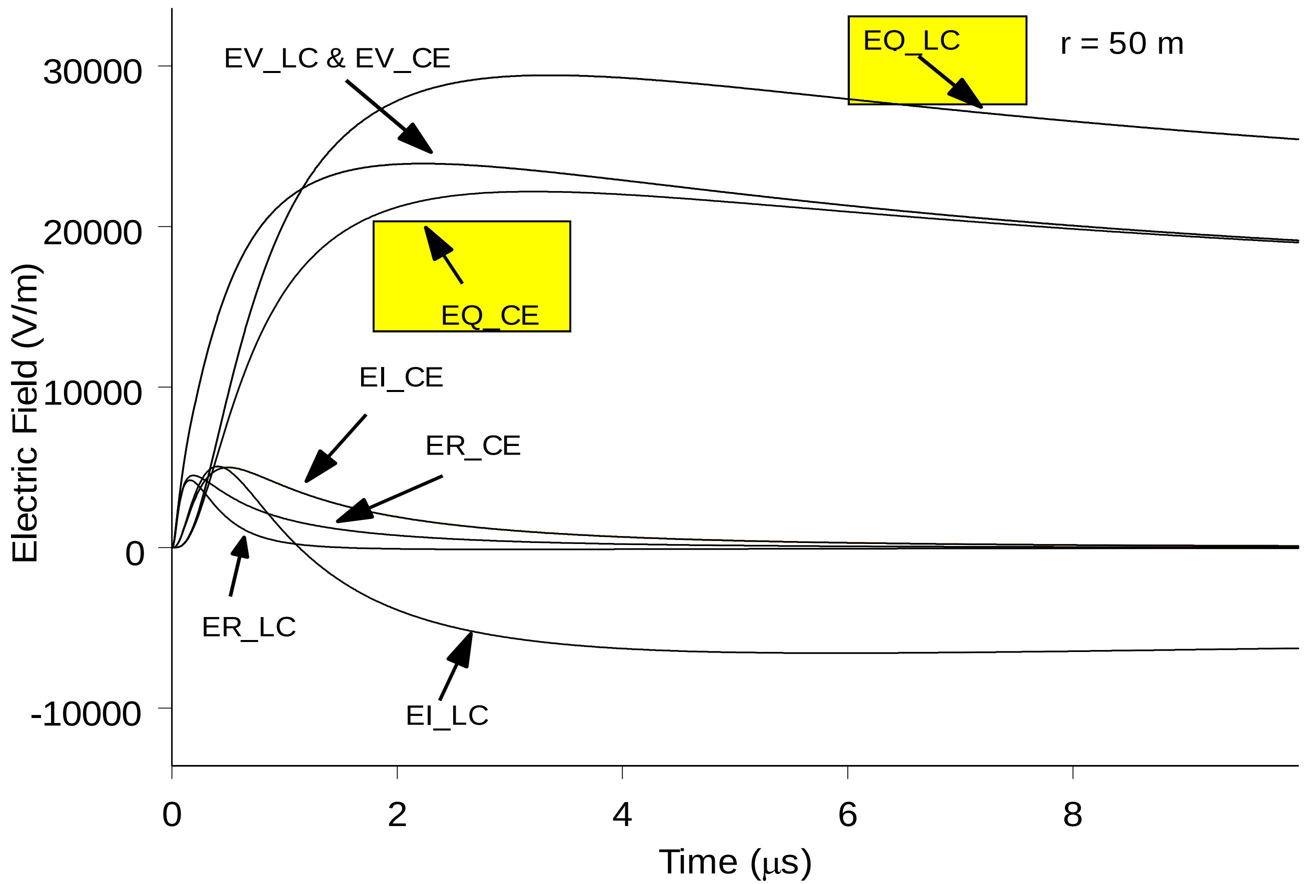
Static $\frac{1}{R^3}$

Induction $\frac{1}{cR^2}$

Radiation $\frac{1}{c^2R}$

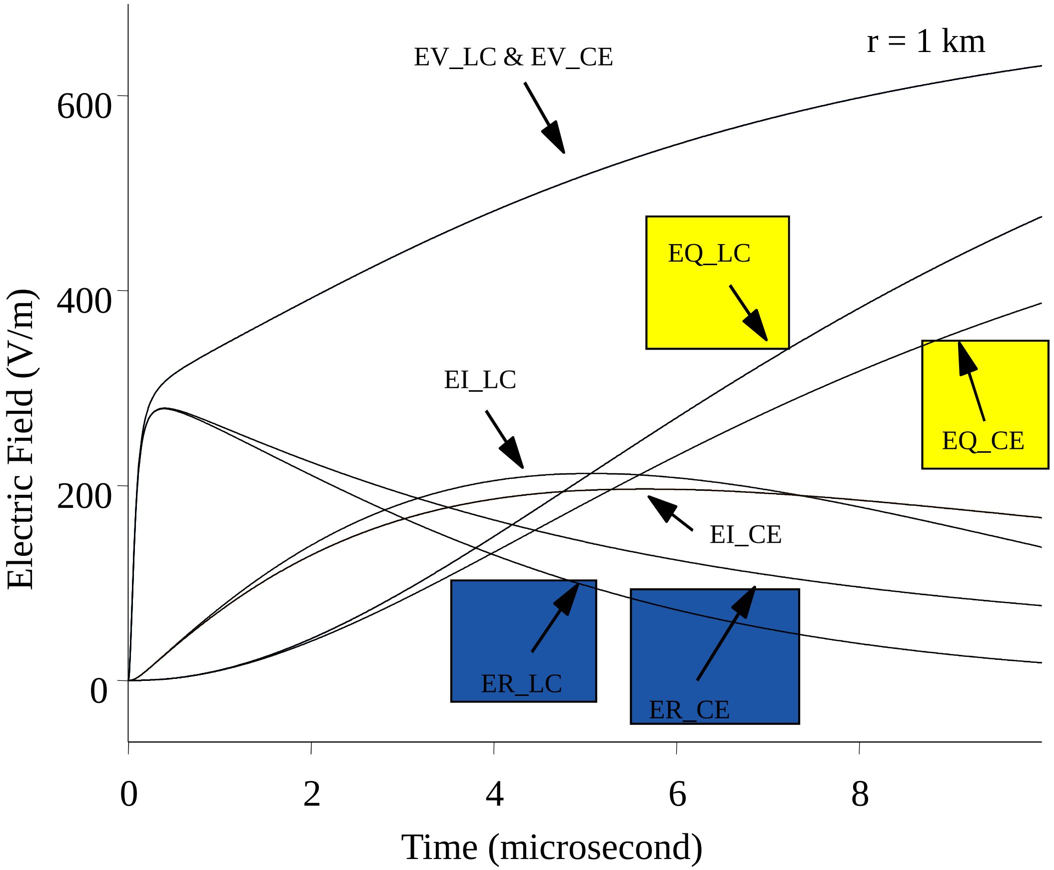
Non-uniqueness of field components (Numerical example) - 1

R = 50 m



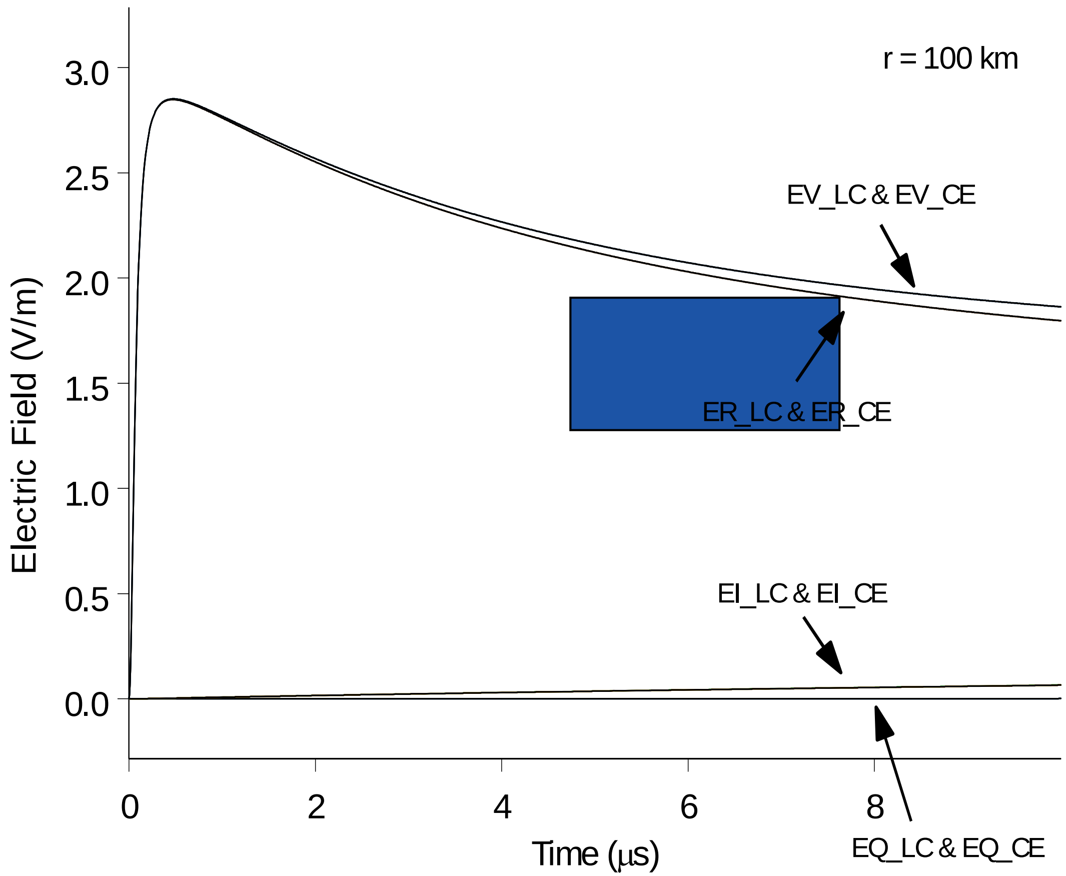
Non-uniqueness of field components (Numerical example) - 2

R = 1000 m



Non-uniqueness of field components (Numerical example) - 3

R = 100 000 m



Electric field at ground plane

(Dipole method)

Both the gradient of the **scalar potential** and the time derivative of the **vector potential** contribute to the **radiation field** term.

Time derivative of the **vector potential** contribute to the induction field term.

(Monopole method)

Radiation term is completely given by the time derivative of the **vector potential**.

Electrostatic and induction terms are given completely by the gradient of the **scalar potential**

No one-to-one correspondence between field components. However, total field is the same

Inferences

- Individual field components - static, induction, and radiation - are not unique
- Total electric field is unique
- Differences between field components are significant at close distances and negligible at far distances
- Caution has to be exercised in interpreting measurement results or in making approximations in calculations

Relation between retarded current and retarded charge

?

$$\frac{\partial \rho^*(z', t - R(z')/c)}{\partial t} = -\left.\frac{\partial i(z', t - R(z')/c)}{\partial z'}\right|_{t - R(z')/c = const.}$$

OR

$$\frac{\partial \rho(z', t - R(z')/c)}{\partial t} = -\frac{\partial i(z', t - R(z')/c)}{\partial z'}$$

Thottappillil , Rakov, Uman (1997)

Relation between two definitions of retarded charge density

$$\rho(z',t-R(z')/c) \quad = \quad \rho^*(z',t-R(z')/c) + \frac{z'-r\cos\theta}{cR(z')} i(z',t-R(z')/c)$$

$$\left(\frac{z'-r\cos\theta}{cR(z')} = -\frac{\partial(R/c)}{\partial z'}\right)$$

Local charge density at retarded time

Local charge density at retarded time as 'seen' by remote observer

(apparent charge density)

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Relation between ‘apparent charge density’ and retarded current

$$\rho(z',t-R(z')/c) = -\frac{d}{dz'}\int_{z'/v+R(z')/c}^{t} i(z',\tau-R(z')/c)\,d\tau$$

Fields at ground in terms of apparent charge density

$$E_z(r,t) = -\frac{1}{2\pi\varepsilon_0}\int_0^{L'(t)} \frac{z'}{R^3(z')}\rho(z',t^*)dz'$$

$$-\frac{1}{2\pi\varepsilon_0}\int_0^{L'(t)} \left(\frac{3}{2}\frac{z'}{cR^2(z')} - \frac{1}{2}\frac{\tan^{-1}(z'/r)}{cr}\right)\frac{\partial\rho(z',t^*)}{\partial t}dz'$$

$$-\frac{1}{2\pi\varepsilon_0}\left(\frac{3}{2}\frac{L'(t)}{cR^2(L')} - \frac{1}{2}\frac{\tan^{-1}(L'(t)/r)}{cr}\right)\rho(L',\frac{L'(t)}{v})\frac{dL'(t)}{dt}$$

$$\vdots \quad -\frac{1}{2\pi\varepsilon_0}\int_0^{L'(t)} \frac{z'}{c^2R(z')}\frac{\partial^2\rho(z',t^*)}{\partial t^2}dz'$$

$$-\frac{1}{2\pi\varepsilon_0}\frac{L'(t)}{c^2R(L')}\frac{\partial}{\partial t}\left[\rho(L'(t),\frac{L'(t)}{v})\frac{dL'(t)}{dt}\right]$$

$$-\frac{1}{2\pi\varepsilon_0}\frac{r^2}{c^2R^3(L')}\rho(L',\frac{L'(t)}{v})\left(\frac{dL'}{dt}\right)^2$$

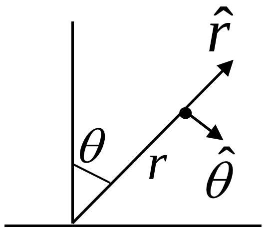
Pulse propagation on a vertical antenna above perfect ground (exact formulation)

- What happens if the return stroke speed is speed of light and if the current travels without any attenuation and dispersion?

- It is proved in Thottappillil et al. (2001) that the exact general expression, with effect of perfect ground included, reduces to

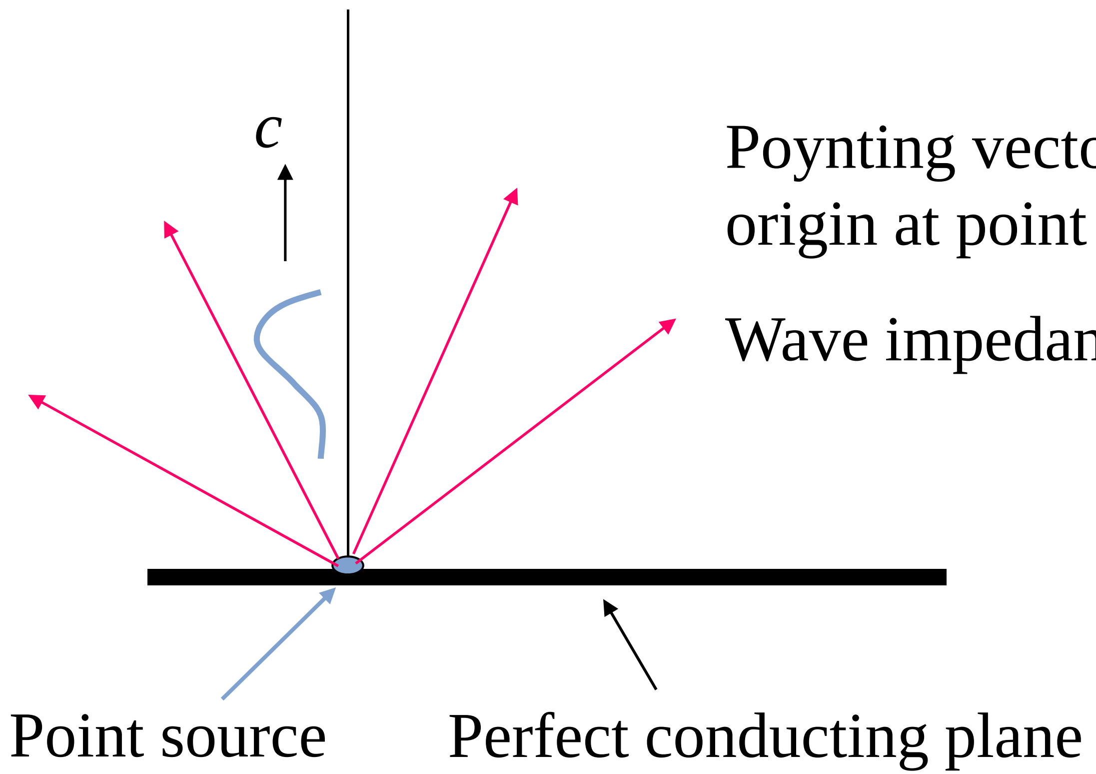
$$\overline{E}(r,\theta,t) = \frac{1}{2\pi\varepsilon_0 cr\sin\theta} i(0,t-r/c)\hat{\theta}, \quad \theta \neq 0$$

$$\overline{B}(r,\theta,t) = \frac{1}{2\pi\varepsilon_0 c^2 r\sin\theta} i(0,t-r/c)\hat{\phi}, \quad \theta \neq 0$$



Cooray and Cooray (2010) also derive same results starting from the fields of a moving point charge.

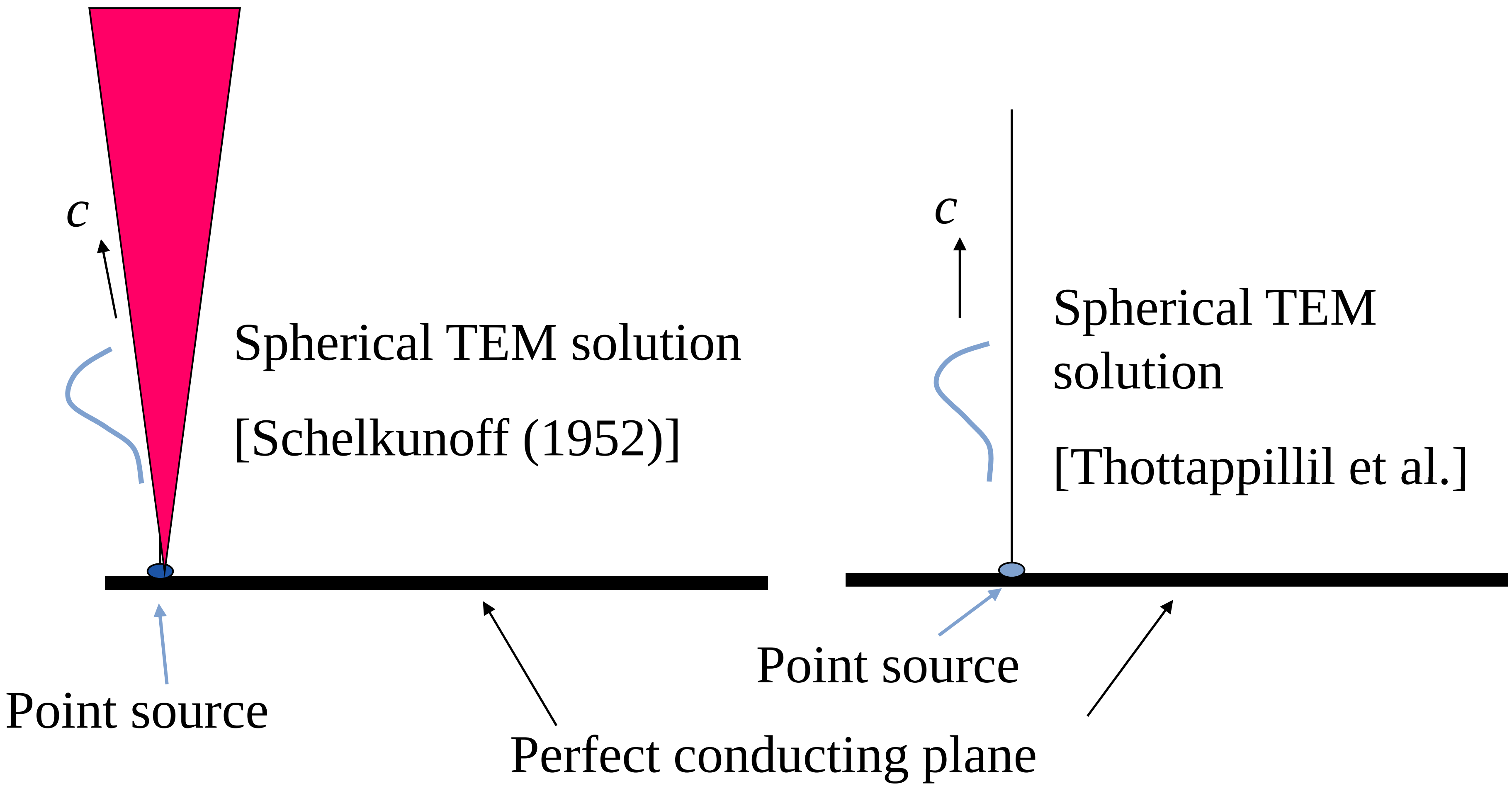
Pulse propagation on a vertical antenna (exact formulation) - continued



Poynting vectors radially-directed with origin at point charge

Wave impedance = 377 Ω

Similarity to the solution of infinite conical antenna



Inferences

- A semi-infinite conducting wire of vanishing radius perpendicular to a conducting plane, all conductors being perfect, support spherical TEM, if the only source is a point source at the bottom of the wire.
- The current released from the point source travels unattenuated with the speed of light.
- The Poynting vector and energy flow is in the radial direction from the source at the bottom of the antenna.
- The wave impedance is the free-space impedance (377 Ω) at all distances from the antenna.

SOME REFERENCES FOR MORE INFORMATION

- [1] Uman, M. A., D. K. McLain, and E. P. Krider, The electromagnetic radiation from a finite antenna, *Am. J. Phys.*, *43*, 33-38, 1975.
- [2] Rubinstein, M., and M.A. Uman, 1989. Methods for calculating the electromagnetic fields from a known source distribution: Application to lightning, IEEE Trans. Electromagn. Comp., 31, 183-189.
- [3] Rubinstein, M. and M. A. Uman, On the radiation field turn-on term associated with travelling current discontinuities in lightning, J. Geophys. Res., 95, 3711-3713, 1990.
- [4] Thottappillil, R., V. A. Rakov, and M. A. Uman, Distribution of charge along the lightning channel: Relation to remote electric and magnetic fields and to return-stroke models, *J. Geophys. Res.*, *102*, 6987-7006, 1997.
- [5] Thottappillil, R., Uman, M.A. and Rakov, V.A. Treatment of retardation effects in calculating the radiated electromagnetic fields from the lightning discharge, J. Geophys. Res., 103, 9003-9013, 1998.
- [6] Thottappillil, R., and V. A. Rakov, On different approaches to calculating lightning electric fields, *J. Geophys. Res.*, *106, 14191-14205*, 2001.
- [7] Thottappillil, R., and V.A. Rakov, On the computation of electric fields from a lightning discharge in time domain, *2001 IEEE EMC International Symposium, Montreal, Canada*, Aug. 13-17, 2001.
- [8] Thottappillil, R., Computation of electromagnetic fields from lightning discharge, Chapter in the book "The Lightning Flash", (ed) V. Cooray, The Institution of Electrical Engineers, London, 2003.
- [9] Thottappillil, R., M.A. Uman, N. Theethayi, Electric and magnetic fields from a semi-infinite antenna above a conducting plane, J. Electrostatics, 61, 209-221, 2004.