The Quandary of Direct Measurement and Indirect Estimation of Lightning Current Parameters

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Abstract: An overview of experimental data on lightning return stroke current obtained by means of instrumented towers and artificially-initiated lightning is presented. Modeling of tower effects and indirect estimation of lightning return stroke current from remote electromagnetic field measurements are also discussed.

1. Introduction

Accurate knowledge of lightning current parameters is essential for the appropriate protection of electric and electronic systems. Experimental data on lightning return stroke currents can be classified into three categories: (1) data obtained using instrumented towers (2) data obtained using triggered lightning which is, to some extent, similar to natural subsequent return strokes, and (3) indirect estimation of lightning return stroke currents from distant electromagnetic field measurements using lightning location systems (LLS). The enormous amounts of data that can be gathered by means of LLS make such systems a promising source of the experimental statistics needed for the development of better standards related to the protection of power and telecommunication systems against lightning.

This paper presents a review of the experimental data on lightning return stroke current obtained during the last few decades and discusses the issue of the indirect estimation of lightning current parameters from distant electromagnetic fields.

2. Direct Measurement

The most complete description to date of lightning return stroke currents at the base of the lightning channel was presented by Berger and co-workers in Switzerland using short instrumented towers. The currents were measured using resistive shunts located at the top of two towers, 70- and 90-m tall, at the summit of Mount San Salvatore in Lugano. The summit of the Mount San Salvatore is 914 m above sea level and 640 m above the level of Lake Lugano, located at the base of the mountain. The measured currents were recorded using high speed cathode-ray oscilloscopes (installed in 1958) with four beams to record currents in both towers and two time deflections with a resolution of 0.5 μs [1]. About 15% of the measurements reported by Berger and co-workers were due to downward-moving stepped leaders. Most discharges to the towers were initiated by upward-moving stepped leaders of both polarities.

2.1 Summary of Berger's data

Figs. 1 and 2, show a compilation of measurements, performed by Berger and co-workers, for lightning initiated by downward-moving leaders.

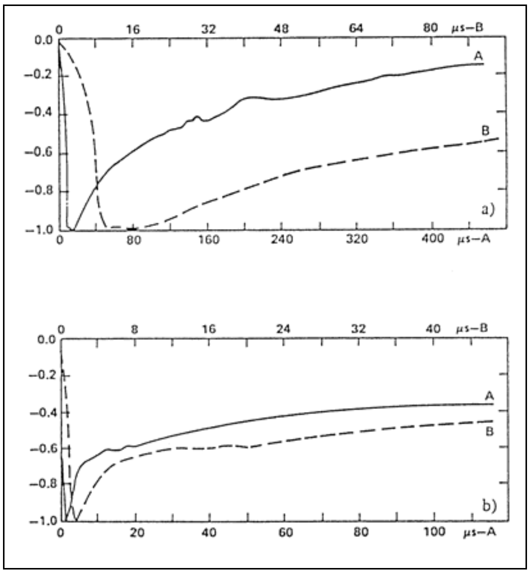

Fig. 1 – Typical, normalized negative return-stroke current wave shapes: (a) First return stroke, (b) Subsequent return stroke (Adapted from [2])

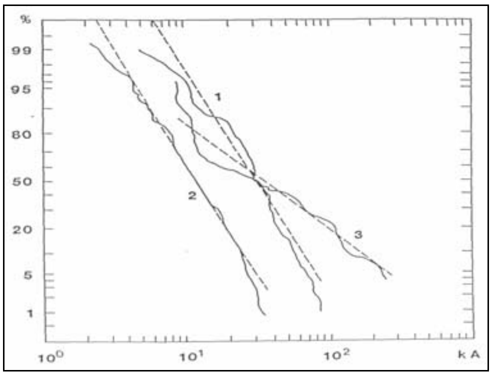

Fig. 2 - Cumulative statistical distributions of return-stroke current peak (solid-line curves) and their log-normal approximations (slanted dashed lines) for (1) negative first strokes, (2) negative subsequent strokes, and (3) positive first strokes as reported by [2]. The vertical scale gives the percentage of peak currents exceeding a given value on the horizontal axis (Adapted from [2])

The waveforms in Fig. 1 correspond to the average of 88 normalized first return stroke waveforms and 76 normalized subsequent return stroke waveforms. They are presented in two time scales, with the label "A" corresponding to a large scale going up to 100 µs and "B" to a smaller span of 40 µs (dashed lines). In Fig. 1, it is possible to observe that, on average, the current associated with subsequent strokes exhibits faster risetimes than those associated with subsequent strokes.

In Fig. 2, the peak current distribution is presented for negative first return strokes, negative subsequent return strokes, and positive return strokes.

The dashed, slanted lines represent a log normal distribution fit to the experimental data for all three cases [3]. The value of the distribution at 50% is around 30 kA for both, first negative and first positive return-strokes. A smaller current value of around 12 kA is observed for the 50 % abscissa for subsequent negative return-stroke current peaks.

Table 1 - Lightning current parameters for downward flashes. (Adapted from [2])

Table 4.1 - Parameter	**Units**	**Sample size**	**Percent Exceeding Tabulated Value**		
			95%	**50%**	**5%**
Peak current (minimum 2 kA)					
Negative first strokes	kA	101	14	30	80
Negative subsequent strokes	kA	135	4.6	12	30
Positive first strokes	kA	26	4.6	35	250
Charge (total charge)					
Negative first strokes	C	93	1.1	5.2	24
Negative subsequent strokes	C	122	0.2	1.4	11
Complete negative flash	C	94	1.3	7.5	40
Impulse charge					
Negative first strokes	C	90	1.1	4.5	20
Negative subsequent strokes	C	117	0.22	0.95	4.0
Positive first strokes	C	25	2.0	16	150
Front duration (2 kA to peak)					
Negative first strokes	µsec	89	1.8	5.5	18
Negative subsequent strokes	µsec	118	0.22	1.1	4.5
Positive first strokes	µsec	19	3.5	22	200
Maximum *di*/*dt*					
Negative first strokes	kA/µsec	92	5.5	12	32
Negative subsequent strokes	kA/µsec	122	12	40	120
Positive first strokes	kA/µsec	21	0.20	2.4	32
Stroke duration (2 kA to half-value)					
Negative first strokes	µsec	90	30	75	200
Negative subsequent strokes	µsec	115	6.5	32	140
Positive first strokes	µsec	16	25	230	2000
Integral (i^2 *dt*)					
Negative first strokes	A^2sec	91	6.0 x 10^3	5.5 x 10^4	5.5 x 10^5
Negative subsequent strokes	A^2sec	88	5.5 x 10^2	6.0 x 10^3	5.2 x 10^4
Positive first strokes	A^2sec	26	2.5 x 10^4	6.5 x 10^3	1.5 x 10^7
Time interval					
Between negative strokes	msec	133	7	33	150
Flash duration					
Negative (including single stroke flashes)	msec	94	0.15	13	1100
Negative (excluding single stroke flashes)	msec	39	31	180	900
Positive (only single flashes)	msec	24	14	85	500

Even if the subsequent return-stroke's current-peak distribution is somewhat lower than half of the first return-stroke current distribution, the shapes of the distributions are similar, as illustrated in Fig. 2 and Table 1.
There is a controversy concerning the front duration and the maximum rate of rise, di/dt in Berger's data. Indeed, the instrumentation used by Berger and co-workers had a limited frequency bandwidth, which may have introduced inaccuracies in their experimental observations, especially for subsequent return strokes [4]. Moreover, it is generally presumed that the obtained statistical data are affected by the presence of the instrumented structure, in that the current peak distribution is biased towards larger values (e.g. [4, 5]).

2.2 Other data obtained using short towers

Other short instrumented towers have been used around the world to measure lightning return stroke parameters. We briefly cite some of them in the next few paragraphs.
Garbagnati, Dellera [6]and co-workers measured currents at the top of 40-m television towers in the 1970s using resistive shunts, located at the top of the towers, and oscillograph recorders. The towers were located on the top of two mountains, each about 900 m above sea level [1] [3] [6]. One of the towers was located in the north of Italy, near Mount San Salvatore (Berger's tower location), and the other tower was located in central Italy. Table 2 summarizes the results of the Italian group for downward flashes.
Eriksson and co-workers measured lightning currents on a 60-m tall tower located above a relatively flat ground in South Africa in the 1970's. The tower was insulated from ground and the lightning current was measured at the bottom via a current transformer and a Rogowski coil.

Table 2 - Return-stroke current parameters measured by Garbagnati and coworkers in Italy for discharges lowering negative charge to ground (Adapted from [3, 6]).

Parameter	Downward	
	First strokes	Subsequent strokes
Sample size	42	33
Peak value (kA)	33	18
Maximum rate of rise (kA/μs)	14	33
Time to crest (μsec) – (3 kA to peak)	9	1.1
Time to half value (μsec)	56	28
Impulse charge (C)	2.8	1.4

More than 50% of the observed flashes were initiated by the usual downward-moving, negatively charged stepped leader. No positive flashes were recorded. Very fast current risetimes were observed in these measurements, not observed in other studies. Table 3 shows values reported by Anderson and Eriksson in 1980 (adapted from [7]).
Other data have been obtained using short towers in Japan [8], in Austria (Gaisberg Tower) [9, 10], in Brazil [11], and in Colombia [12-14].

Table 3 - Return-stroke current parameters measured by Eriksson and coworkers in South Africa for natural subsequent strokes lowering negative charge to ground (Adapted from [7]).

Parameter	Subsequent natural strokes		
	95%	50%	5%
Sample size	114		
Peak value (kA)	4.9	12	29
10 - 90% average of current steepness (kA/μs)	3.3	15	72
10 - 90% time duration (μsec)	0.1	0.6	2.8

2.3 Data from tall towers

Lightning return stroke currents measured on the 540-m high Ostankino tower in Moscow represent the first measurements of currents performed simultaneously at three different heights along a tower. The three current sensors were installed at 533, 272 and 47 m above ground level as reported by [15]. The lightning return-stroke current observations present different wave shapes at the three observation points (Fig. 4). The differences are presumably due to reflections produced at the tower discontinuities during the initial lightning current propagation to ground.

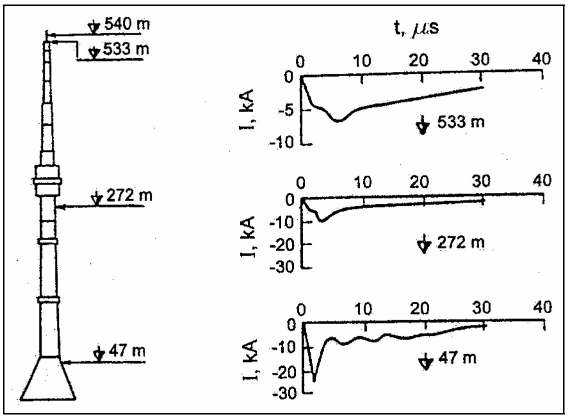

Fig. 4 - Sample of return stroke current waveshape of upward negative lightning, recorded at three different locations in the Ostankino tower in Moscow (Adapted from [15]).

From the three wave shapes presented in Fig. 4, we can see that the largest "absolute peak" amplitude appears at the lower observation point (about 22 kA). It seems that, at the point of discontinuity between the bottom of the tower and the grounding impedance, there is a positive reflection of current that adds to the initial return stroke current. This positive reflection from the bottom is clearly discernible at the other two locations some microseconds later. The fact that the peak amplitude of the current measured at 533 m (8 kA) is smaller than the peak amplitude at 272 m (10 kA) indicates that a negative reflection coefficient can be associated with the top of the tower. This coefficient represents the discontinuity between the tower and the "equivalent" impedance of the lightning channel.

Rakov reports a median peak value for currents measured at 47 and 533 m of 18 and 9 kA, respectively. He suggests that the effective grounding impedance of the tower is much smaller than its characteristic impedance and that this is appreciably lower than the equivalent impedance of the lightning channel [15].

Studies on lightning striking the CN Tower (553-m high) in Toronto, Canada, the tallest free-standing structure in the world, have been performed and reported by the "CN Tower Lightning Studies Group (CNTLSG)" since 1978 (e.g. [16-19]). The lightning return-stroke current derivatives striking the CN tower are measured by two inductive Rogowski coils located at 509 and 474 m height.

A sample of lightning return-stroke current observed at the CN Tower in 1999 is presented in Fig. 5. Lightning return-stroke currents and current derivatives observed at the CN Tower have been found to exhibit multiple reflections produced at the tower discontinuities. The observed currents and current derivatives are therefore "contaminated" by these reflections.

The waveshapes of current in Fig. 5 exhibit a positive reflection arriving around 3.6 microseconds after the first current maximum. This propagation time corresponds to a round-trip time from the tower top to ground, confirming that this reflection was produced at the lower discontinuity level between the tower-bottom and the grounding impedance. The positive value of the reflection implies a positive ground reflection coefficient. The observed positive reflection is less pronounced for the sensor located closer to the top of the tower. This is similar to the observations at the Ostankino tower, suggesting a negative top reflection coefficient.

However, comparing the wave shapes for the observed currents in Figs. 4 and 5, we can see that the currents observed on the CN Tower exhibit a more complex structure than those of the Ostankino Tower. This is probably due to the architecture of the CN Tower which presents a number of features that act as discontinuities to the current (see Fig. 2.11), as suggested by Shostak [20].

A more complete study of reflections produced in the CN Tower data was recently presented by Shostak and co-workers (see [20-24]).

The 168-m tall Peissenberg tower located near Munich, Germany, on a ridge 250 m above the surrounding open ground and 950 m above sea level, was used from 1978 until 1999 to study lightning currents and their associated electromagnetic fields [25]. The tower had two current measurement systems installed, respectively, at approximately 167 m and 13 m. The systems were able to measure return stroke currents and their derivatives. During the time of exploitation of the tower, only one stroke of a downward negative flash (cloud-to-ground lightning) was recorded by the system. The majority of the strokes recorded at the Peissenberg tower were produced by upward flashes (ground-to-cloud lightning), with negative or positive polarity. Fig. 6b presents waveforms of return-stroke currents measured simultaneously at the bottom and top of the tower in which the "contamination" of the current by multiple reflections is clearly distinguishable.

The current wave shapes in Fig. 6 exhibit a higher peak value for the current observed at the bottom of the tower.

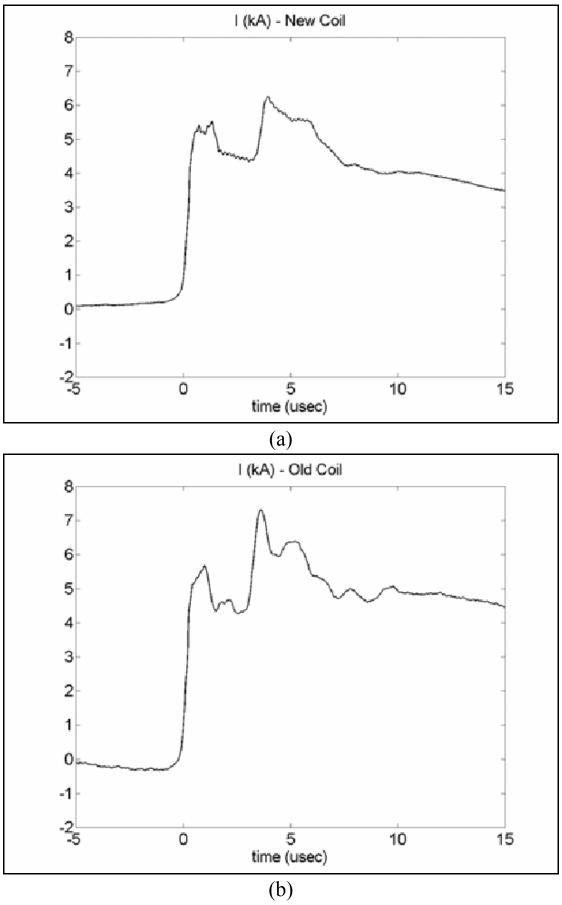

Fig. 5 - Sample of a lightning return-stroke current observed at (a) 509 and (b) 474 m height at the CN Tower in Toronto (adapted from [26])

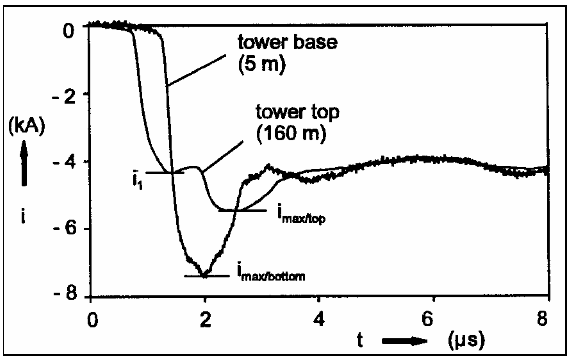

Fig. 6 - Comparison of a lightning return-stroke current recorded at the Peissenberg tower top and bottom (adapted from [25])

A 250-m tall telecommunication tower was instrumented in St. Chrischona, near Basel, in Switzerland, with two current loop antennas at 248 and at 175 m, and an additional current probe at the top. The tower was located at the summit of a hill 500 m above sea level. The two current derivative systems as well as the current probe were used over a period of 5 years to record lightning return stroke current wave shapes impacting the tower [27, 28].

The 200-m high Fukui tower in Japan was also used to measure lightning return-stroke currents and their associated electromagnetic fields at the Fukui thermal power plant on the coast of the Sea of Japan. Two coaxial shunt resistors (2 mΩ, 10 mΩ) were installed at the top of the tower [29]. It was found that the measured current was affected by reflected waves at the ground and the top of the tower. Fig. 7 presents a schematic representation of the installation of the Fukui tower and electromagnetic field recording system.

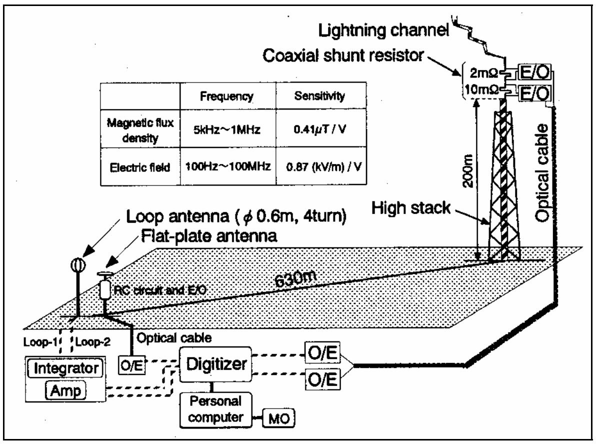

Fig. 7 - Configuration of lightning stroke current and electromagnetic field observation systems at the Fukui thermal power plant (adapted from [29])

It is worth mentioning one of the first experimental studies of lightning currents obtained at the top of 380 m high Empire State Building in 1935 reported in [30]. The current was observed using the crater-lamp oscillograph, magnetic links and, at about 780-m distance, rotating cameras. The majority of the oscillograms recorded indicated negative currents, produced by upward-moving stepped leaders. McEachron [30] was the first to discover the existence of upward-moving leaders. The leader current merged smoothly into a continuous current flow between cloud and building without the occurrence of a return stroke. In about half of these discharges, subsequent return-stroke current peaks initiated by downward-moving dart leaders followed the initial discharge stage. The maximum current recorded was 58 kA, associated with a positive stroke. The upward-moving stepped leader was found to have an average step length of 8.2 m [1, 3].

2.4. Data from triggered lightning

The possibility of initiating lightning artificially by ground-based activity was apparently first investigated in the early 1960's. The first triggered lightning events were produced in 1960 by launching small rockets trailing thin, grounded wires from research vessels off the Florida coast. The first triggered lightning over ground was accomplished in 1973, at Saint Privat d'Allier (SPA) in France. In the following decades, a number of triggered-lightning programs have been developed in different countries, e.g. Saint Privat d'Allier in France, Kahokugata in Japan, Langmuir laboratory in New Mexico, Kennedy Space Center in Florida, Okushishiku in Japan, Fort McClellan in Alabama and Camp Blanding in Florida [31]. This technique provided additional information concerning the return-stroke lightning current at the base of the channel as well as the associated electromagnetic fields. Rocket-triggered lightning flashes are usually upward-moving leader initiated and their characteristics are very similar to natural subsequent strokes.

Concerning the characterization of return-stroke current waveforms for classical triggered lightning, the summarized observations in Florida and France presented by [31] are shown in Table 4.

Table 4 - Characterization of return-stroke current peak and peak derivative from classical triggered lightning experiments in KSC-Florida and SPA-France (Adapted from [31]).

Location	Years	Sample size	Current peak (kA)		Current derivative peak (kA/μs)	
			Median	STD	Median	STD
KSC Florida	1985-1991	305/ 134	12.1	9.0	91.4	97.1
SPA France	1986, 1990-1991	54/47	9.8	5.6	36.8	25.4

As seen in Table 4, the median values for the current are 12.1 and 9.8 kA in the USA and France, respectively. These median values differ by approximately 20%. Note the similarity of the value for the median current measured in Florida with the average value reported by Berger et al. (Table 1).

The rocket-and-wire technique is now frequently used for artificial initiation of lightning from natural thunderclouds in the context of lightning research. Leader/return stroke sequences in triggered lightning are similar to subsequent strokes in natural downward lightning, although the initial processes in classical triggered lightning are distinctly different from the first leader/return stroke sequence in natural downward lightning. Notwithstanding these differences, triggered lightning is a valuable research tool to investigate natural lightning. Indeed, the results of triggered lightning experiments have provided a number of insights into various lightning processes that would have been virtually impossible to obtain from direct studies of natural lightning due to its random occurrence in space and time.

One must be aware, however, of the differences between some of the properties of artificially initiated lightning when compared to its natural counterpart. Triggered lightning typically occurs in cloud conditions under which the discharge is unlikely to start independently. In addition, there is contamination of the lower portion of the lightning channel by metallic wire residue. Moreover, the channel terminates at a triggered-lightning facility having specific geometrical and electrical characteristics [32]. Triggered flashes have been reported to differ from natural lightning flashes in that they exhibit a larger number of strokes per flash, a higher dart leader velocity, and a shorter inter-stroke interval duration [33].

Table 5 summarizes statistical results for the current peak and risetime obtained using various instrumented towers around the world, as well as using the triggered lightning technique.

3. Inclusion of the Strike Object in Return Stroke Models

3.1 Introduction

The interaction of lightning with tall strike objects has recently attracted a considerable attention of lightning researchers e.g. [15, 21, 27, 34-37]. For this reason, some of the return stroke models, initially developed for the case of return strokes initiated at ground, were extended to take into account the presence of a vertically-extended strike object e.g., [23, 29, 38-48]. In some of these models, it is assumed that a current pulse $i_o(t)$ associated with the return-stroke process is injected at the lightning attachment point both into the strike object and into the lightning channel, e.g., [23, 29, 39, 40, 43-47, 49]. The upward-moving wave propagates along the channel at the return-stroke speed v as specified by the return-stroke model. The downward-moving wave propagates at the speed of light along the strike object, assumed to be a lossless uniform transmission line characterized by constant non-zero reflection coefficients at its top and its bottom. As noted by [41], the assumption of two identical current waves injected into the lightning channel and into the strike object implies that their characteristic impedances are equal to each other. This assumption makes the models not self-consistent in that (1) there is no impedance discontinuity at the tower top at the time of lightning attachment to the tower, but (2) there is one when the reflections from ground arrive at the tower top.

Recently, Rachidi et al [50] presented an extension of the so-called engineering return stroke models, taking into account the presence of a vertically-extended strike object, which does not employ the assumption that identical current pulses are launched both upward and downward from the object top. The extension is based on a distributed-source representation of the return-stroke channel [51, 52], which allows more general and straightforward formulations of these models than the traditional representations implying a lumped current source at the bottom of the channel.

3.2 Engineering return stroke models including the presence of a vertically-extended strike object

The general equations for the spatial-temporal distribution of the current along the lightning channel and along the strike object have been derived [50]:

$$i(z',t)=\left[P(z'-h)\,i_o\left(h,t-\frac{z'-h}{v^*}\right)-\rho_t\,i_o\left(h,t-\frac{z'-h}{c}\right)+\right.$$
$$\left.+(1-\rho_t)(1+\rho_t)\sum_{n=0}^{\infty}\rho_g^{n+1}\rho_t^{n}\,i_o\left(h,t-\frac{h+z'}{c}-\frac{2nh}{c}\right)\right]u\left(t-\frac{z'-h}{v}\right)$$
$$\text{for } h<z'<H_0 \qquad (1)$$

$$i(z',t)=(1-\rho_t)\sum_{n=0}^{\infty}\left[\rho_t^{n}\rho_g^{n}\,i_o\left(h,t-\frac{h-z'}{c}-\frac{2nh}{c}\right)+\right.$$
$$\left.+\rho_t^{n}\rho_g^{n+1}\,i_o\left(h,t-\frac{h+z'}{c}-\frac{2nh}{c}\right)\right]u\left(t-\frac{h+z'}{c}-\frac{2nh}{c}\right)$$
$$\text{for } 0\le z'\le h \qquad (2)$$

In (1) and (2), h is the height of the tower, ρt and ρg are the top and bottom current reflection coefficients for upward and downward propagating waves, respectively, H_0 is the height of the extending return stroke channel, c is the speed of light, $P(z')$ is a model-dependent attenuation function, $u(t)$ the Heaviside unit-step function, v is the return-stroke front speed, and v^* is the current-wave speed.

Expressions for $P(z')$ and v^* for some of the most commonly used return-stroke models are summarized in Table 6, in which λ is the attenuation height for the MTLE model and H_{tot} is the total height of the lightning channel.

Equations (1) and (2) are based on the concept of 'undisturbed current' $i_0(t)$, which represents the 'ideal' current that would be measured at the tower top if the current reflection coefficients at its both extremities were equal to zero.

It is assumed that the current reflection coefficients ρ_t and ρ_g are constant. In addition, any upward connecting leader and any reflections at the return stroke wavefront [23] are disregarded.

Table 6 - $P(z')$ and v^* for different return-stroke models (Adapted from [58]).

Model	$P(z')$	v^*
BG	1	∞
TCS	1	$-c$
TL	1	v
MTLL	$1-z'/H_{tot}$	v
MTLE	$\exp(-z'/\lambda)$	v

Table 5 - Comparison of return-stroke current peaks in downward and upward flashes measured in instrumented towers, and rocket triggered lightning effectively transporting negative charge to ground (Adapted from [53]).

Location	**Height (m)**	**Location**	**Sample size**	**Ipeak (50%) (kA)**	**Risetime (μs)**	**Sensor Location**
CN Tower - Toronto (1992-2001) ([26, 54]) Upward flashes	553	Canada	387	5.06[a] 7.19[b]	0.64[f]	Top (474 m)
Ostankino Tower - Moscow (1984) ([15]) Negative upward flashes at 533 m Negative upward flashes at 47 m	540	Russia	58 76	9 18	- -	533 m 47 m
Empire State Building – USA (1952) ([15]) Upward flashes	410	USA	84	10	-	Top
Fukui thermal power plant (1989-1994) ([55]) Upward flashes type A (strong luminosity) Upward flashes type B (low luminosity)	200	Japan	22 33	33 (23.5[c]) 3.4	1 – 2[c]	Top
Peissenberg Tower–Germany (1992/98) ([25]) Negative upward flashes, α-component[j] Negative upward flashes, β-component[j]	168	Peissenberg at 167 m	89 68	3.63[b] 7.97[b]	- -	Top
Japan Transmission Towers (1994/97) ([8]) Negative downward flashes, 1st stroke at top	40-140	Japan	36	39	4.5[g]	Top
Gaisberg Tower (Austria) [56] Downward negative, first strokes Downward negative, subsequent strokes Upward negative, α-component[j] Upward negative, β-component[j]	100	Austria		14[i] 2.91[i] 2.94[i] 8.57[i]	- -	Top
Mount Saint Salvatore Tower-CH ([2]) Negative downward flashes, 1st stroke Negative downward flashes, 2nd stroke Upward flashes	70-90	Switzerland	101 135 70	30 12 10	5.5[d] 1.1[d] -	Top
South Africa Tower (1980) ([7, 31]) Negative downward flashes	60	South Africa	114	12[b]	0.6[h]	Bottom
Morro do Cachimbo Tower-Brazil ([57]) Negative downward strokes, 1st stroke Negative downward strokes, subs. stroke	60	Brazil	31 59	40.4[a] 45.3[b] 16.3[b]	5.6[h] 0.7[h]	Bottom
Italy TV Towers ([3, 6]) Negative downward flashes, 1st stroke Negative downward flashes, 2nd stroke Negative upward flashes, 1st stroke Negative upward flashes, 2nd stroke	40	Italy	42 33 61 142	33 18 7 8	9[e] 1.1[e] 4[e] 1.3[e]	Top
ROCKET-TRIGGERED FLASHES:						
Depasse (1994) ([15, 31])	5.0-20	Florida (KSC), USA	305	12.1	-	Bottom
Depasse (1994) ([15, 31])	5.0-20	France	54	9.8	-	Bottom
Fisher and coworkers in USA (1993) ([15])	≤ 7.8	Florida (KSC) & Alabama	45	13	-	Bottom

[a] First peak of the current
[b] Absolute peak of the current
[c] Values reported by [29]
[d] Front duration defined as the time interval between the 2 kA point on the front and the first peak, the time resolution of the system was 0.5 μs.
[e] Time to crest defined between 3 kA to peak.
[f] Risetime to wavefront peak
[g] The time resolution of the system was 100 ns ([8])
[h] time interval between instants corresponding to 10% and 90% of first current peak
[i] 1 kA threshold
[j] α-components: pulses superimposed on the initial continuous current, β-components: pulses following a period of no current in the channel.

3.3 Far field-current relations

The determination of the peak return stroke current from remotely measured electric or magnetic fields considerably facilitates the collection of lightning return stroke current data without having to instrument towers or trigger the lightning artificially, and without the inherent relative inefficiency associated with those methods. This is especially true nowadays because of the widespread use of lightning location systems. Indeed, such systems are already used to provide also estimates of lightning current parameters (e.g. [59, 60]).

The theoretical estimation of return stroke currents from remote electromagnetic fields depends on the adopted return stroke model. Expressions relating radiated fields and return stroke channel base currents have been derived for various 'engineering' return stroke models in which the lightning channel is terminated at ground (e.g. [61]).

We will present in this section the analytical relationship between lightning currents and far electromagnetic fields, for various engineering models and taking into account the presence of an elevated strike object [62].

The general expressions for the vertical electric field and the azimuthal magnetic field from a vertical antenna above a perfectly conducting ground, for an observation point at ground level (see Fig. 2), are given by [63]

$$E_z(r,t)=\frac{1}{2\pi\varepsilon_o}\left[\int_0^H \frac{2z'^2-r^2}{R^5}\int_{R/c}^{t} i(z',\tau-R/c)d\tau dz' + \int_0^H \frac{2z'^2-r^2}{cR^4} i(z',t-R/c)dz' - \int_0^H \frac{r^2}{c^2R^3}\frac{\partial i(z',t-R/c)}{\partial t}dz\right] \quad (3)$$

$$H_\varphi(r,t)=\frac{1}{2\pi}\left[\int_0^H \frac{r}{R^3} i(z',t-R/c)dz' + \int_0^H \frac{r}{cR^2}\frac{\partial i(z',t-R/c)}{\partial t}dz'\right] \quad (4)$$

where H is the height of the return stroke wavefront as seen by the observer, r is the horizontal distance between the channel and the observation point, and R is the distance between a single dipole located at a height z' above ground and the observation point ($R=\sqrt{r^2+z'^2}$).

Let us consider here only the radiated electromagnetic field. For distant observation points, neglecting the static and induction components of the electric field, and considering $R \cong r$ and $r >> H$, the general expression for the electric and magnetic fields for an observation point located at ground level reduces to (Fig. 9)

$$E_z^{far}(r,t)=-\frac{1}{2\pi\varepsilon_o c^2 r}\int_0^H \frac{\partial i(z',t-r/c)}{\partial t}dz' \quad (5)$$

$$H_\varphi^{far}(r,t)\cong\frac{1}{2\pi cr}\int_0^H \frac{\partial i(z',t-r/c)}{\partial t}dz' \quad (6)$$

Introducing the general expressions for the spatial-temporal distribution of the current, given by equations (1) and (2), into equations (5) and (6), and after appropriate mathematical manipulations, we obtain

$$E_z^{far}(r,t)=E_z^{\mathrm{rs}}(r,t)+E_z^{\mathrm{eso}}(r,t)+E_z^{\mathrm{turn-on}}(r,t) \quad (7)$$

$$H_\varphi^{far}(r,t)=H_\varphi^{\mathrm{rs}}(r,t)+H_\varphi^{\mathrm{eso}}(r,t)+H_\varphi^{\mathrm{turn-on}}(r,t) \quad (8)$$

in which

- $E_z^{\mathrm{rs}}(r,t)$ and $H_\varphi^{\mathrm{rs}}(r,t)$ are the electric and magnetic radiation fields due to the main return stroke pulse, that is, the first term on the right hand side of Equation (1), given by

$$E_z^{\mathrm{rs}}(r,t)=-\frac{1}{2\pi\varepsilon_o c^2 r}\int_h^H P(z'-h)\cdot \frac{\partial}{\partial t}\left[i_o(h,t-\frac{z'-h}{v^*})u(t-\frac{z'-h}{v})\right]dz' \quad (9)$$

$$H_\varphi^{\mathrm{rs}}(r,t)=\frac{1}{2\pi cr}\int_h^H P(z'-h)\cdot \frac{\partial}{\partial t}\left[i_o(h,t-\frac{z'-h}{v^*})u(t-\frac{z'-h}{v})\right]dz' \quad (10)$$

- $E_z^{\mathrm{eso}}(r,t)$ and $H_\varphi^{\mathrm{eso}}(r,t)$ are the electric and magnetic fields resulting from the contribution of the multiple-reflection process along the elevated strike object (eso), including the contribution of pulses transmitted into the channel. They are given by [64]

$$E_z^{\mathrm{eso}}(r,t+r/c)=-\frac{1}{2\pi\varepsilon_o c^2 r}(1-2\rho_t)i_o(h,t)-\frac{(1-\rho_t)}{2\pi\varepsilon_o cr}\sum_{n=0}^{\infty}\left\{\begin{array}{l}(\rho_g-1)\rho_g^n\rho_t^n\, i_o\left(h,t-\frac{(2n+1)h}{c}\right)\\ +2\rho_g^{n+1}\rho_t^{n+1}\, i_o(h,t-\frac{2(n+1)h}{c})\end{array}\right\} \quad (11)$$

$$H_\varphi^{\mathrm{eso}}(r,t+r/c)=\frac{1}{2\pi r}(1-2\rho_t)i_o(h,t)+\frac{(1-\rho_t)}{2\pi\varepsilon_o cr}\sum_{n=0}^{\infty}\left\{\begin{array}{l}(\rho_g-1)\rho_g^n\rho_t^n\, i_o\left(h,t-\frac{(2n+1)h}{c}\right)\\ +2\rho_g^{n+1}\rho_t^{n+1}\, i_o(h,t-\frac{2(n+1)h}{c})\end{array}\right\} \quad (12)$$

- and, finally, $E_z^{\mathrm{turn-on}}(r,t)$ and $H_\varphi^{\mathrm{turn-on}}(r,t)$ are the so-called 'turn-on' electric and magnetic field terms [65], associated with the current discontinuity at the return stroke wavefront. Indeed, the current distribution associated with the extended models exhibits a discontinuity at the return stroke wave front. This discontinuity arises from the fact that, due to back and forth reflections in the tower, current pulses are transmitted back up into the return stroke channel at the top of the tower. These transmitted pulses, which are assumed to travel at the speed of light, catch up with the return stroke wave front traveling at a lower speed and, since no current is allowed to flow in the leader region above the front [65], the current stops there abruptly. This discontinuity needs to be carefully treated when calculating the radiated electromagnetic field through an additional 'turn-on' term in the electromagnetic field equations, given by

$$E_z^{\text{turn-on}}(r,t) = -\frac{I_{front}(H)r^2}{4\pi\varepsilon_o c^2 R^3}\frac{1}{\frac{1}{v}+\frac{H}{cR}} - \frac{I_{front}(H')r^2}{4\pi\varepsilon_o c^2 R'^3}\frac{1}{\frac{1}{v}+\frac{H'}{cR'}} \quad (13)$$

$$H_\varphi^{\text{turn-on}}(r,t) = \frac{I_{front}(H)r}{4\pi c R^2}\frac{1}{\frac{1}{v}+\frac{H}{cR}} - \frac{I_{front}(H')r}{4\pi c R'^2}\frac{1}{\frac{1}{v}+\frac{H'}{cR'}} \quad (14)$$

where *H*' and *R*' are, respectively, the length of the return stroke channel image and the distance of the wavefont image from the observer, both calculated considering the propagation time delay at the observation point [65]. I_{front} is the amplitude of the current discontinuity at the wavefront.

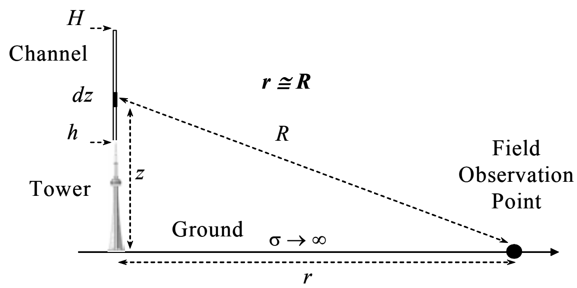

Fig. 9 - Geometry for the far-field calculation.

It is interesting to note that, the second and the third terms of the electromagnetic fields, namely $E_z^{\text{eso}}(r,t)$, $H_\varphi^{\text{eso}}(r,t)$ and $E_z^{\text{turn-on}}(r,t)$, $H_\varphi^{\text{turn-on}}(r,t)$ are independent of the model. The only model-dependent term is $E_z^{rs}(r,t)$ for the electric field, and $H_\varphi^{rs}(r,t)$ for the magnetic field. Tables 7 and 8 summarize the specific expressions for $E_z^{rs}(r,t)$ and $H_\varphi^{rs}(r,t)$ developed for various models (e.g. [61]).

Table 7 - Expressions to calculate $E_z^{rs}(r,t)$ for different return stroke models, far-field conditions. ($k = 1 + v/c$)

Model	$E_z^{rs}(r,t)$
BG	$-\frac{v}{2\pi\varepsilon_o c^2 r}\left[i_o(h,t)+t\frac{di_o(h,t)}{dt}\right]$ (15)
TL	$-\frac{v}{2\pi\varepsilon_o c^2 r} i_o(h,t)$ (16)
TCS	$-\frac{v}{2\pi\varepsilon_o c^2 r}\left[k i_o(h,kt) - i_o(h,t)\right]$ (17)
MTLL	$\frac{dE_z^{rs}(r,t)}{dt} = -\frac{v}{2\pi\varepsilon_o c^2 r}\left[\frac{di_o(h,t-r/c)}{dt} - \frac{v}{H} i_o(h,t-r/c)\right]$ (18)
MTLE	$\left[\frac{E_z^{rs}(r,t+r/c)}{\lambda}+\frac{1}{v}\frac{dE_z^{rs}(r,t+r/c)}{dt}\right] = -\frac{1}{2\pi\varepsilon_o c^2 r}\frac{di_o(h,t)}{dt}$ (19)

Table 8 - Expressions to calculate $H_\varphi^{rs}(r,t)$ for different return stroke models, far-field conditions. ($k = 1 + v/c$)

Model	$H_\varphi^{\text{rs}}(r,t)$
BG	$\frac{v}{2\pi c r}\left[i_o(h,t)+t\frac{di_o(h,t)}{dt}\right]$ (20)
TL	$\frac{v}{2\pi c r} i_o(h,t)$ (21)
TCS	$\frac{v}{2\pi c r}\left[k i_o(h,kt) - i_o(h,t)\right]$ (22)
MTLL	$\frac{dH_\varphi^{rs}(r,t)}{dt} = \frac{v}{2\pi c r}\left[\frac{di_o(h,t-r/c)}{dt} - \frac{v}{H} i_o(h,t-r/c)\right]$ (23)
MTLE	$\left[\frac{H_\varphi^{rs}(r,t+r/c)}{\lambda}+\frac{1}{v}\frac{dH_\varphi^{rs}(r,t+r/c)}{dt}\right] = \frac{1}{2\pi c r}\frac{di_o(h,t)}{dt}$ (24)

Additionally, it is important to note that the BG and the TCS model exhibit an inherent discontinuity at the return stroke wavefront. This discontinuity gives rise to a turn-on term which is already included in the main pulse ($E_z^{rs}(r,t)$ and $H_\varphi^{rs}(r,t)$) contributions.

3.3 Particularization for the TL model

Bermudez et al. [64] have derived expressions relating the lightning return stroke current peak to the distant electric or magnetic field peak, considering two cases: (1) 'tall' structures, for which the round-trip propagation time from top to bottom within the tower (*2h/c*) is greater than the current zero-to-peak risetime t_f; in which case the current transmitted into the tower reaches its peak before the arrival of any ground reflections (none of the reflections overlap with it); and, (2) electrically-short structures, for which the round-trip propagation time is much shorter than the lightning return stroke current wavefront t_f; in which case, we can neglect propagation delays along the tower.

3.3.1 'Tall' strike object ($t_f < 2h/c$)

In this case, it is shown that Equations x reduce to

$$E_{z\,peak} = -\frac{v}{2\pi\varepsilon_o c^2 r}\left[1+\frac{c}{v}\left(1-2\rho_t\right)\right] I_{o\,peak} \qquad (25)$$

$$H_{\varphi\,peak} = \frac{v}{2\pi c r}\left[1+\frac{c}{v}\left(1-2\rho_t\right)\right] I_{o\,peak} \qquad (26)$$

where $I_{o\,peak}$ is the first peak of the undisturbed current $i_o(h,t)$.

It is important to note that the undisturbed current $i_o(h,t)$ is different from the actual current pulse injected from the channel into the tower top. It would be therefore more appropriate to express the electromagnetic field peaks as a function of the current transmitted into the tower, for which experimental data are usually available. To do that, one needs to express the undisturbed current peak $I_{o\,peak}$ as a function of the peak of the current transmitted into the tower, I_{peak}. Under the current conditions of tall towers ($t_f < h/c$), these two quantities are simply related by

$$I_{peak} = (1-\rho_t) I_{o\,peak} \qquad (27)$$

where ρ_t is given by $\rho_t = \dfrac{Z_t - Z_{ch}}{Z_t + Z_{ch}}$.

Introducing (27) into (25) and 26), we obtain

$$E_{z\,peak} = -\frac{v}{2\pi\varepsilon_o c^2 r} k_{tall}\, I_{peak} \qquad (28)$$

$$H_{\varphi\,peak} = \frac{v}{2\pi c r} k_{tall} I_{peak} \qquad (29)$$

where k_{tall} is given by

$$k_{tall} = \frac{1+\left(1-2\rho_t\right)c/v}{1-\rho_t} \qquad (30)$$

The above expressions show that the enhancement effect of the tower can be quantified through the factor k_{tall}. Because of the condition $t_f < 2h/c$ imposed on the current, equations (29) and (30) are independent of the structure's height *h* and of the ground reflection coefficient ρ_g.

It is also important to note that, as the grounding impedance of the tower is generally noticeably smaller than its own characteristic impedance and since the characteristic impedance is, in turn, appreciably lower than the equivalent impedance of the lightning channel (e.g. [15, 43]), the current reflection coefficient at the ground ρ_g is positive and the top reflection coefficient ρ_t is negative. Thus, the factor k_{tall} in equations (29) and (30) is greater than 1, implying that the presence of the strike object enhances the electric and magnetic field peaks in comparison to return strokes initiated at ground level.

In Section 4, we will compare the results obtained using the derived expressions for current and field peaks associated with tall towers with simultaneous measurements of return stroke current, and electric and magnetic fields associated with lightning strikes to the CN Tower in Toronto.

3.3.2 Electrically-short strike object ($t_f >> h/c$)

Let us now consider the special case when the strike object is electrically short. This would be the case, for instance, when lightning is initiated artificially from short plataforms, or when the upward-connecting leaders are present at ground (assuming that they can be represented by a short vertical transmission line), or for very long-front pulses.

In this case, propagation along the tower can be neglected and closed-form expressions can be derived for the spatial-temporal distribution of the current along the strike object and along the channel. For this case, the far-field current relation reduces to [64]

$$E_{z\,peak} = -\frac{v}{2\pi\varepsilon_o c^2 r} k_{short}\, I_{peak} \qquad (31)$$

$$H_{\varphi\,peak} = \frac{v}{2\pi c r} k_{short}\, I_{peak} \qquad (32)$$

where k_{short} is given by

$$k_{short} = \frac{1+\frac{c}{v}\rho_{ch-g}}{1+\rho_{ch-g}} \qquad (33)$$

In terms of impedances, (33) can be written as

$$k_{short} = \frac{Z_{ch}(1+\frac{c}{v}) + Z_g(1-\frac{c}{v})}{2Z_{ch}} \qquad (34)$$

Equations (31) and (32) will reduce to the well-known expressions for the classical TL model (ground-initiated return stroke)[1], when the reflection coefficient $\rho_{ch\text{-}g}$ equals 0 or, equivalently, when $Z_{ch} = Z_g$. This consideration, that $\rho_{ch\text{-}g}$ equals 0, implies that no current reflections will occur between the lightning channel and the ground, as assumed in the classical TL model. Thus, (31) and (32) can be considered as extended forms of (35) and (36) for return strokes initiated at ground level, in which the reflections at ground are incorporated in the TL model.

Equations (31) and (32) can be employed to find the return stroke electric and magnetic field peak amplitudes associated with lightning impacting the ground. These equations take into account the impedance discontinuity between the lightning channel and the grounding impedance. Comparing equations (31) and (32) with equations (35) and (36), we can see that including the grounding condition will result in an enhancement of far fields, which is quantified through the factor k_{short}.

The question of whether or not the effect of such impedance discontinuity for short structures has been experimentally observed deserves some discussion. Short strike objects are used in triggered lightning launching structures. As discussed in [40], Willett et al. [66] have investigated the accuracy of the classical TL model for estimating return stroke current parameters from remote electromagnetic fields from triggered lightning. Leteinturier et al. [67] have made similar measurements and both [66] and [67] have concluded that the observed fine structure of the field was not consistent with a single-pulse propagating upward assumed by the classical TL model. Their study suggests that considering two wavefronts traveling upward and downward from the junction point, improves substantially the agreement between measured and inferred propagation speeds.

Note that this enhancement factor k_{short} is generally much smaller than the factor associated with tall towers k_{tall}. Indeed, considering the ideal case when the grounding impedance is 0 and the return stroke speed $v=c/2$, the enhancement factor k_{short} becomes equal to 1.5.

Note, finally, that the above far-field-current expressions do not include the 'turn-on' term (equations (13) and (14) associated with the discontinuity of the current at the return stroke wavefront.

3.3 Comparison with experimental data

Simultaneous measurements of return stroke current and its associated electric and magnetic fields at two distances related with lightning strikes to the 553-m tall Toronto Canadian National (CN) Tower were performed during 2000 and 2001 [64, 68]. The data have been used to test equations (19) and (20). For the comparison, we will assume a return stroke speed of $v = 1.2\text{x}10^8$ m/s as reported by [69], and a top reflection coefficient of $\rho_t = -0.366$ inferred from experimental data by [17].

Figures 10 and 11 present scatter plots of the experimentally measured peak electric field vs. the peak current and the peak magnetic field vs. the peak current, respectively. We have also plotted in those figures the predictions of the equations (28) and (29) as well as those of equations (35) and (36), which disregard the presence of the elevated strike object.

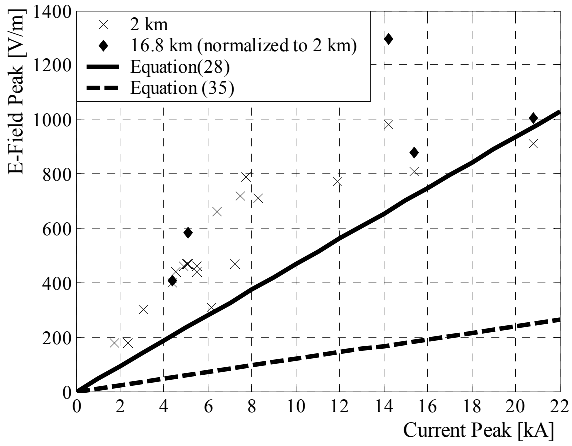

Fig. 10 - Electric field peak as a function of return stroke current peak. Comparison between experimental data and computed results using Equation (28) and Equation (35). The values for the field peaks at 16.8 km have been normalized to those at 2 km assuming a $1/r$ dependence. (Adapted from [64])

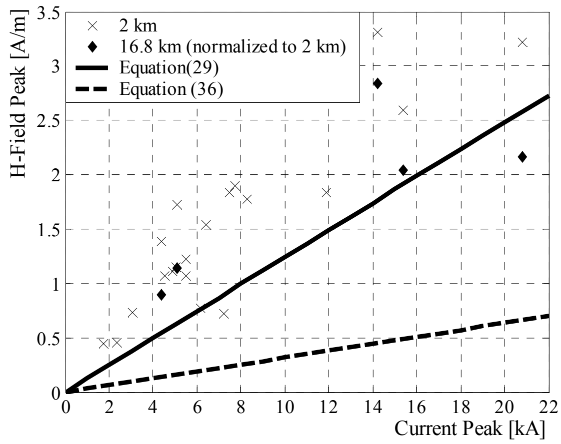

Fig. 11 - Magnetic field peak as a function of return stroke current peak. Comparison between experimental data and computed results using Equation (29) and Equation (36). The values for the field peaks at 16.8 km have been normalized to those at 2 km assuming a $1/r$ dependence. (Adapted from [64])

It can be seen that the values for the peak electromagnetic field predicted by equations (35)-(36), which assume that the return stroke is initiated at ground level, are

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[1] These equations are, respectively,

$$E_{z\,peak} = -\frac{v}{2\pi\varepsilon_o c^2 r} I_{peak} \qquad (35)$$

$$H_{\varphi\,peak} = \frac{v}{2\pi c r} I_{peak} \qquad (36)$$

underestimated by a factor of about 5. Adjusting the return stroke speed in equations (35)-(36) to obtain good agreement leads to speeds much higher than the speed of light. It can also be seen that equations (19)-(20) yield a better estimate of the electromagnetic field peak. However, the field values seem to be still underestimated by the new equations. The differences between theoretical predictions and experimental data can be explained, at least in part, by [64]

- the uncertainty in the adopted values for ρ_t and v;
- assumptions in the theoretical model and experimental errors;
- the fact that, at 2 km from the channel, where most of the electric and magnetic fields were measured, not only the radiation term, but also the electrostatic term (for the E-field) and the induction term (for both E- and H-fields) contribute to the peak value [70];
- the effect of reflections at structural discontinuities of the CN Tower [43, 71];
- the field enhancement effect of the buildings on which the electromagnetic field sensors were installed. In [72], such an enhancement for a 20-m to 30-m building was theoretically estimated and experimentally obtained in [73], and found that it can be significant.

The enhancement of the fields due to the structures on which the sensors are installed requires more study. Further, this enhancement may be a function of frequency and it could therefore distort the lightning fields.

4. Indirect Estimation

4.1 Introduction

The problem of the remote determination of lightning return stroke currents has been getting attention in the past few years due to the widespread use of lightning location systems (LLS). Indeed, in addition to the event time and strike point position, the LLS data can provide estimates for the lightning return-stroke peak current [74]. If a reliable relation between remotely measured quantities and the lightning currents can be obtained, these systems could be used to obtain the statistical data needed for the development of local standards related to the protection of power and telecommunication systems against lightning [75].

The most common method currently employed by modern LLS is to infer the currents from measured distant radiation fields (electric or magnetic) produced by lightning return strokes. This method is appealing since it uses physical quantities that are easily obtained with today's instrumentation and since it can be applied over large geographical areas. However, a number of factors limit the accuracy of these estimates, yielding 20-30% error at best for individual discharges [76]. There is an inherent difficulty in extracting lightning current parameters from LLS-measured electromagnetic fields since unknown parameters – such as the return stroke speed – along with possible current reflections (in the case of strikes to a tall object) within the strike object affect the lightning current inferred from distant electromagnetic fields [40, 44]. In this section, we will discuss various approaches to the estimation of parameters of peak current distributions from parameters of measured peak field distributions.

4.2 Deterministic empirical approach

Estimates of lightning peak currents from measured lightning electromagnetic fields are obtained by way of empirical [77-80] equations relating the electromagnetic field and the lightning current. The empirical equations are based on triggered-lightning experimental data. Based on 28 triggered lightning events consisting of return stroke current, two-dimensional return stroke speed and electric field, Rakov et al. [79] proposed the following regression equation relating the return stroke current peak I and the electric field peak E measured at distance D:

$$I = 1.5 - 0.03ED \tag{27}$$

where I is in kA and taken as negative, E is positive and in V/m, and D is in km.

4.3 Deterministic model-based approach

Expressions relating far electromagnetic fields and associated return stroke currents at the channel base have been derived in the literature for various lightning return stroke models [61]. The use of such relations permits the estimation of channel base currents of return strokes, and the estimation of not-directly-measurable parameters of the models [61]. Tables 7 and 8 summarize the equations relating far electric and magnetic fields to channel base currents according to various return stroke current models. It can be seen that all these equations involve a certain number of parameters, in particular the return stroke speed v, which, in most practical cases, are unknown. In other words, to infer the lightning current from its associated distant electric or magnetic field, one has to assume the value for the return stroke speed. This speed changes, however, from one stroke to another and, as a result, exhibits significant statistical variation (e.g. [81, 82]). It follows from the far field-channel base current relations that an error in the estimation of return stroke speed would result in practically the same amount of error in the inferred channel-base current (see also [83]). In fact, distant electric and magnetic field peaks are determined as much by return-stroke speeds as they are by return-stroke current peaks.

4.4 Stochastic approach

Recently, adopting the TL model and considering the return stroke current peak I, the return stroke speed v, and the distant electric field peak E as random variables, Rachidi et al. [84] have derived the following expressions relating statistical parameters (mean value and variance) of the Electric field to those of the return stroke current and speed. These expressions read

$$\eta_E \cong \frac{1}{2\pi\varepsilon_o c^2 r}\eta_v \eta_I + \frac{1}{2\pi\varepsilon_o c^2 r}\rho_{vI}\sigma_v\sigma_I \quad (37)$$

and

$$\sigma_E^2 \cong \left(\frac{1}{2\pi\varepsilon_o c^2 r}\eta_I\right)^2 \sigma_v^2 + \left(\frac{1}{2\pi\varepsilon_o c^2 r}\eta_v\right)^2 \sigma_I^2 + \frac{1}{\pi\varepsilon_o c^2 r}\frac{1}{2\pi\varepsilon_o c^2 r}\eta_I\eta_v\rho_{vI}\sigma_I\sigma_v \quad (38)$$

in which η_E, η_v and η_I, σ_E, σ_v and σ_I, are the mean values and variances of the peak electric field, return stroke speed and peak return stroke current, respectively.

Neglecting any correlation between current peak and return stroke speed (see [85] for a discussion of this issue), equations (37) and (38) become, respectively:

$$\eta_E \cong \frac{1}{2\pi\varepsilon_o c^2 r}\eta_v \eta_I \quad (39)$$

$$\sigma_E^2 \cong \left(\frac{1}{2\pi\varepsilon_o c^2 r}\eta_I\right)^2 \sigma_v^2 + \left(\frac{1}{2\pi\varepsilon_o c^2 r}\eta_v\right)^2 \sigma_I^2 \quad (40)$$

It is interesting to observe that equation (39) has the same mathematical form as Equation (35), where the values for *E*, *v*, and *I* are simply replaced by the respective mean values η_E, η_v and η_I. This result gives to some extent a theoretical justification to the use of LLS to infer statistical parameters of lightning current from measured fields alone. In other words, although it seems impossible to obtain reasonably accurate estimate of return stroke current from distant field measurements for a single event without prior knowledge of the return stroke speed, this could be done statistically, in terms of mean value and standard deviation, using field measurements acquired by LLS, provided that statistical data for the return stroke speed are available from independent measurements and are not varying much within the area covered by the LLS.

The derived equations have been validated using simultaneous measurements of return stroke current, electric field at 5 km, and return stroke speed associated with triggered lightning return strokes and reported by Willett et al. in [86]. Table 9 summarizes values for return stroke current peak, electric field (essentially radiation) peak at 5 km, and photographically-measured two-dimensional return stroke speed for 17 events. The corresponding mean values, standard deviations, and correlation coefficients are given in Table 10.

Let us assume that the channel-base current is unknown. Inserting statistical parameters associated with field and return stroke speed into equations (37) and (38), the following values can be calculated for the return stroke current

$\eta_I = 18.5$ kA and $\sigma_I = 7.0$ kA

which are in very good agreement with values (see Table 11) determined from measurements presented in Table 10.

Similarly, starting from statistical parameters for current and return stroke speed, one can obtain from equations (37) and (38) the following values for the electric field

$\eta_E = 113.4$ V/m and $\sigma_E = 57.5$ V/m

which, again, are in excellent agreement with the values in Table 10 determined from data presented in Table 9.

Note that, since the correlation coefficients ρ_{Iv} and ρ_{Ev} are very small, the use of equations (39) and (40) instead of more general equations (37) and (38) does not introduce significant errors. This is an important observation since it suggests that no knowledge (usually unavailable) of the correlation coefficients is necessary.

Table 9 – Simultaneous measurements of channel-base current peak, electric field peak at 5 km and return stroke speed associated with 17 rocket-triggered lightning return strokes (adapted from [86]).

Flash/Stroke	I (kA)	E at 5 km (V/m)	$v_{2\text{-}D}$ (10^8 m/s)
8705/1	8.2	76	1.8
8705/3	7.7	64	1.6
8705/5	10.3	80	1.7
8705/6	11.6	84	1.9
8715/9	33	206	1.4
8715/10	15.6	88	1.4
8725/1	20.3	109	1.6
8725/2	16.7	84	1.5
8725/3	43	196	1.7
8725/5	11.7	60	1.4
8726/2	26.9	174	1.2
8726/3	16.4	95	1.4
8726/4	22.4	143	1.4
8728/10	26.9	144	1.4
8728/11	16.1	88	1.4
8732/1	18.0	126	1.5
8732/2	16.8	97	1.6

Table 10 – Statistical parameters for the data presented in Table 1. (Adapted from [84])

Parameter	I (kA)	$v_{2\text{-}D}$ (10^8 m/s)	E (V/m)
Min. value	7.7	1.2	60
Max. value	43	1.9	206
Mean value	18.9	1.5	112.6
Standard deviation	9.3	0.18	45.1
ρ_{Iv}	0.0144		
ρ_{Ev}		0.0268	

5. Conclusions

Currently available data on the lightning current comes from direct measurements using instrumented towers or triggered lightning. In addition, estimates of lightning current parameters can also be achieved indirectly from measurements of lightning electromagnetic fields.

The overview presented in Table 5 shows that lightning current data from instrumented towers have been measured by a large number of research groups under very different geographical conditions using different measuring equipment and recording different sets of parameters. In order to use and compare all these sets of data, a clear definition of the different parameters to be measured, a precise specification of the measurement system, and a standardized presentation of the data are needed.

More recent experimental data of lightning current and current-derivative obtained at the top of tall telecommunications towers have clearly shown the effect of reflections at the top and at the bottom of the tower on the measured current. As a consequence, some of the return stroke models, initially developed for the case of return strokes initiated at ground, were extended to take into account the presence of a vertically-extended strike object.

The indirect estimation of lightning current parameters from measured fields has grown in importance in the last years due to the extensive use of the lightning location systems (LLS). The basic aim of such systems is to provide density maps of lightning flashes. However, more recently, LLS have also been used to estimate lightning current parameters using empirical formulae. Recent studies have shown that, for an assumed return stroke model, a statistical estimation of the current peak value (e.g. in terms of mean values and standard deviations) is possible. We have seen, in addition, that for the Transmission Line (TL) model, the equation permitting to infer the mean value of the return stroke current from the mean values of field and speed has the same functional form as the well-known TL current – far field relationship. This result gives, to some extent, a theoretical justification to the use of LLS to infer statistical parameters of lightning current from measured fields alone, assuming that the return-stroke speed does not vary much within the area covered by the LLS.

Since lightning frequently strikes tall metallic objects such as Franklin rods, radio towers, etc., the presence of such elevated strike objects is to be taken into account when inferring the current from lightning fields.

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